

# Unveiling the Failure of Positive Selection\*

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## Abstract

In the dynamic screening problem between an uninformed seller and a privately informed buyer, theory suggests that the presence of the buyer's outside option leads to a significant surplus for the seller. However, the result relies heavily on multiple layers of positive selection reasoning. In this study, we conduct a two-round bargaining experiment to empirically test whether participants demonstrate positive selection in their belief updates. Our experiment employs finite price alternatives, simplifying the analysis by focusing on first-, second-, and third-order positive selections. We find that only a few subjects adhere to the equilibrium prediction reasoning, with the majority failing even at the first-order positive selection. Consequently, the average seller payoffs fall substantially short of the theoretical benchmark.

**Keywords:** Positive Selection, Outside Options, Laboratory Experiments

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# 1 Introduction

We investigate a bargaining game involving an uninformed seller and a privately informed buyer with a known prior distribution of buyers’ willingness to pay (types). Building upon this framework, [Board and Pycia \(2014\)](#)—hereinafter referred to as BP—introduces an outside option for buyers, which could represent an alternative product offered by a third party. Buyers with lower types are more inclined to exercise this outside option and exit the market rather than engaging in negotiations with the seller. Consequently, the remaining demand pool primarily consists of higher-type buyers. This *positive selection* emerging among the remaining demand pool motivates the seller to charge a monopoly price that maximizes the expected profit over the prior distribution of buyer types. This pricing strategy expedites the bargaining outcomes without any delays. Thus, BP’s theoretical analysis reveals that the introduction of the buyer’s outside option significantly benefits the seller. Notably, this finding remains robust, as even a minuscule positive value for the buyer’s outside option generates qualitatively similar bargaining outcomes.<sup>1</sup>

Positive selection serves as a mechanism that allows a monopoly seller in the market to overcome its lack of commitment power and convert a significant portion of consumer surplus into profit. Consequently, BP’s findings have important implications for market design and regulatory policies across various markets, such as durable-good monopolies, sequential auctions, and lemon markets, which are centered around the dynamic screening problem. If the goal of the market designer is to maintain consumer surplus, BP’s results suggest that it is sufficient to prevent buyers from accessing any outside options.<sup>2</sup> However, this policy implication appears to contradict the conventional wisdom that restricting monopoly power generally fosters market competition and increases consumer surplus. Therefore, it is crucial to empirically validate the existence of positive selection before discussing its policy implications. This justification supports our approach of employing controlled laboratory experiments to obtain empirical evidence.

The primary purpose of our laboratory experiment is to examine whether the experiment participants exhibit positive selection in their belief updates. Our experiment is not merely comparing the predictions of [Board and Pycia \(2014\)](#) with our experimental data: By considering a simple

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<sup>1</sup>[Catonini \(2022\)](#) further reinforces BP’s findings by proving that the equilibrium strategy is the uniquely strongly rationalizable strategy.

<sup>2</sup>When there is no outside option available, the bargaining game reverts to the standard dynamic screening problem, where the Coase Conjecture holds true. In this case, the uninformed seller does not benefit from intertemporal price discrimination among different buyer types. The remaining buyers at any given price are more likely to be of lower types, leading to a phenomenon known as *negative selection* in the demand pool. In the absence of commitment power, the seller responds by gradually reducing the offering price over time. Anticipating this price decline, even a high-type buyer tends to delay their purchase, prompting the seller to lower the price even in the early stages to encourage any purchase. As a result, the seller effectively charges the lowest price consistently, earning the minimum possible expected profit in equilibrium. The concept of negative selection has been theoretically explored and confirmed by researchers such as [Fudenberg et al. \(1985\)](#) and [Gul et al. \(1986\)](#). In our companion paper ([Chang et al., 2023](#)), we examine the treatment effect of the outside option by comparing environments with and without an outside option.

two-round bargaining environment with three discrete types, we can clearly separate out higher-order reasoning from the lower-order ones. Our main interests are to examine the first-, second-, and third-order positive selections. Roughly speaking, the first-order positive selection implies that when a buyer has a low type, he should take the outside option immediately. The second-order positive selection implies that the low- and middle-type buyers do not have an incentive to delay so the seller believes that in the second round, it is more likely to encounter a high-type buyer. The third-order positive selection implies that the high-type buyer does not have an incentive to delay as well, and the seller, who correctly updates her belief, charges the monopoly price that maximizes the expected earnings of a stage game. It is a natural consequence that no delay occurs if these orders of rationality hold. Our simple experimental design also allows us to exclude other compounding factors. For example, by restricting the price options to only three, we enforce the effect of other-regarding preferences to be minimal, and the seller's choices are free from ad-hoc tie-breaking rules.

We found that a substantial fraction (41.77%) of the first-round price offers are rejected and then leads to a delay of the negotiation to the second round, while theory predicts that buyers should either accept the offer or exercise the outside option immediately. Examining the seller's posterior belief after the rejection of the first-round offer, we found that only a small proportion of the sellers updated their belief in line with the idea of positive selection: About 37% of the posterior beliefs can be rationalized in the first-order positive selection, about 25% of them can be rationalized in the second-order positive selection, and only about 7% of them can be understood as a result of the third-order positive selection. Worse yet, the posterior belief through the third-order positive selection coincides with the prior, and the second-round price offer confirms that more than half of those who reported the equilibrium posterior happened to be the naïve players who stick to the prior belief. As a result, few experiment participants behaved in a way that the BP's logic unfolds. Also, the theory presented in BP and the simplified version of ours predict that the presence of an outside option leads to a substantially large profit for the seller. In our experiment, the seller's average profit turned out to be much lower than they could have attained in theory, which sharply contradicts the main prediction from the positive selection.

The main contribution of this paper is to present and utilize a way to decipher which level of positive selection reasoning fails. This is beyond merely examining the validity of the theoretical predictions of the asymmetric information bargaining game with an outside option. We found a clear rejection of the theoretical predictions, and our experimental design allows us to enunciate the drivers of the discrepancies between experimental data and theoretical predictions. We hope our method of experimentally stripping down the several layers of rational reasoning to be used in various contexts whose equilibrium predictions are derived from multiple thought processes.

The rest of this paper is organized as follows. In the following subsection, we discuss the closely related literature. Section 2 describes the theoretical environment. Section 3 describes the

experimental design and procedure. The results are reported in Section 4. Section 5 concludes.

## 1.1 Literature Review

Positive selection has garnered significant attention in the literature. Board and Pycia (2014) demonstrates that introducing the buyer’s outside option, even if its value is arbitrarily close to zero, challenges the Coase conjecture by enabling the seller to earn substantially higher profits than predicted by the conjecture. The role of outside option and positive selection in bargaining is further investigated in Hwang and Li (2017), Chang and Lee (2022), and Fanning (2023), among others.

Similarly, Tirole (2016) shows that a principal can achieve the outcome of a profit-maximizing mechanism without commitment power in a wide range of dynamic screening problems, contradicting the Coase conjecture that implies the principal earns the least profit. These findings highlight the divergent effects of positive selection and negative selection. While negative selection generally harms the principal’s interests, positive selection results in the optimal outcome for the principal. We contribute to the literature on the dynamic screening problem by providing experimental evidence.

In the sense that our experimental design allows us to strip down the first-, second-, and third-order positive selection reasoning, we also contribute to the literature on the analysis and identification of higher-order rationality. Kneeland (2015) proposes an explicit design of experiments called “ring games” to identify higher-order rationality. By presenting four different payoff tables with different decision timing, the ring game asks participants to reveal their levels of reasoning. Meanwhile, players in our experiment encounter the same game all the time and we elicit their levels of reasoning by observing the reported beliefs.

Many bargaining situations in the real world extend beyond two rounds of offers. Thus, although the two-round experiment in the current paper is best suited to test whether and to what extent subjects exhibit positive selection in their belief updates, it is not proper to test how positive selection (or failure of it) affects the bargaining dynamics and eventual outcomes in the real world based on our experiment. Our companion paper (Chang et al., 2023) complements this limit. To answer whether positive selection indeed makes differences in bargaining dynamics and outcomes, Chang et al. (2023) study infinite-horizon bargaining with and without an outside option.

## 2 Theoretical Background: Two-Round Model

### 2.1 Model

**Environment** Consider a two-round ( $t = 1, 2$ ) version of BP’s bargaining game. A seller (he) and a buyer (she) negotiate the price of an indivisible good. The buyer’s type (the value of the good)

$v$  is private information, which is drawn from a *finite* support  $V \subset (0, \infty)$  before the negotiation begins. For any  $v' \in V$ , let  $q(v') = \mathbb{P}\{v = v'\} \in (0, 1)$  denote the probability that the buyer's true type is  $v'$ . The buyer also has an outside option. The outside option is available in both rounds, and its value is type-independent and given as  $w > 0$ .<sup>3</sup> Each buyer type's net-value is defined as  $u(v) := v - w > 0$  and also assumed to be strictly positive. The seller's valuation of the good is normalized as zero; the seller has no outside option.

In the first round  $t = 1$ , the seller first makes a price offer  $p_1$  from the *finite* set of feasible offers  $\mathcal{P} \subset \mathbb{R}_+$ ; the assumption imposed to  $\mathcal{P}$  will be discussed shortly. Then, the buyer chooses to accept this offer, exercise the outside option, or reject both the offer and the outside option to delay the final decision to the next round. In the second round  $t = 2$ , if ever happens to occur, the seller again makes a price offer  $p_2 \in \mathcal{P}$ , and then, the buyer chooses to accept this offer, exercise the outside option, or reject both the offer and the outside option.

In addition to the buyer's and the seller's decisions as described above, Nature may step in with probability  $\epsilon \in [0, 1]$  at the end of the first round. In the case that Nature intervenes, it overrides any decision by the buyer in the first round and forces the negotiation to be delayed to the second round.<sup>4</sup> When the second round occurs, the seller cannot distinguish whether it was the buyer's voluntary decision or Nature's intervention that brought the negotiation to the second round; the seller updates his posterior beliefs, being aware of both possibilities.

The bargaining game ends when the buyer accepts the seller's offer or exercises the outside option (without Nature's intervention) in either round. For each possible outcome of the game, the buyer and the seller respectively obtain the following final payoffs:

$$\begin{array}{ll} \delta^{t-1}(v - p_t) & \text{and } \delta^{t-1}p_t & \text{if the buyer accepts } p_t \text{ in round } t \in \{1, 2\}, \\ \delta^{t-1}w & \text{and } 0 & \text{if the buyer exercises the outside option in round } t \in \{1, 2\}, \end{array}$$

where  $\delta \in (0, 1)$  represents the common discounting factor. Both the seller and the buyer obtain zero payoff if the buyer rejects both  $p_2$  and the outside option in the second round.

**Assumptions** We impose several assumptions to the set of feasible prices  $\mathcal{P}$ . First, we assume

$$\mathcal{P} \cap \{u(v) : v \in V\} = \emptyset. \tag{A1}$$

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<sup>3</sup>We assume that outside options are type-independent for the sake of simplicity. The analysis in this section remains valid even with type-dependent outside options. The original BP's model also allows the outside option to be fully type-dependent, including the case with a type-independent outside option as a special case.

<sup>4</sup>Note that we consider both cases  $\epsilon = 0$  and  $\epsilon > 0$ . Taking the possibility of Nature's intervention into account, if the buyer chooses to accept  $p_1$  or exercise the outside option in the first round, the bargaining still can be delayed to the second round with probability  $\epsilon$ . The bargaining is delayed to the second round for sure if the buyer chooses to delay in the first round.

In other words, there is no price offer in  $\mathcal{P}$  that would make any buyer type exactly indifferent between accepting it and exercising the outside option. We impose this assumption to avoid the multiplicity of equilibria that could arise from the indeterminacy of the buyer's tie-breaking rule.

To state the second assumption, define first  $\bar{p}(v) := \max\{p \in \mathcal{P} : u(v) = v - w > p\}$  as the *highest* price among ones acceptable to each buyer type  $v \in V$ . In addition, let  $\Delta > 0$  denote the largest gap between  $u(v)$  and  $\bar{p}(v)$  among all buyer types:

$$\Delta := \max_{v \in V} |u(v) - \bar{p}(v)|.$$

We assume that the set  $\{p \in \mathcal{P} : u(v) = v - w > p\}$  is non-empty and thus  $\bar{p}(v)$  is well-defined for each  $v$ . Furthermore,  $\Delta$  is assumed to be sufficiently small such that

$$\Delta < \frac{1 - \delta}{\delta} (1 - \epsilon) w. \quad (\text{A2})$$

This assumption holds, for example, if  $\mathcal{P} = \{kp_0 : k \in \{0\} \cup \mathbb{N}\}$  where  $p_0 \approx 0$  is a sufficiently small monetary unit.

**Strategies and Equilibrium** For each buyer type  $v \in V$ , let  $\sigma_v(A|p_1)$  denote the probability that the buyer of type  $v$  accepts  $p_1$  in the first round.  $\sigma_v(O|p_1)$  denotes the probability that the buyer of type  $v$  exercises the outside option in the first round, conditional on the event that the seller offers  $p_1$ . Finally,  $\sigma_v(D|p_1)$  denotes the probability of choosing to delay conditional on the same event. The buyer's decision in the second round is straightforward: for any  $p_2 \in \mathcal{P}$  offered by the seller, each buyer type  $v$  chooses to accept  $p_2$  iff  $u(v) > p_2$ . Thus, without any loss, we omit the buyer's behavioral strategy in the second round in describing her (equilibrium) strategy. Let  $\sigma = \langle \sigma_v : \{A, O, D\} \times \mathcal{P} \rightarrow [0, 1] \rangle_{v \in V}$  generically denote the buyer's mixed strategy.

The seller's pure strategy chooses a sequence of price offers  $(p_1, p_2) \in \mathcal{P} \times \mathcal{P}$ . Let  $\tau$  generically denote a mixed strategy of the seller (a probability distribution over  $\mathcal{P} \times \mathcal{P}$ ). The seller also forms posterior beliefs regarding the buyer's type in the second round. We will denote by

$$\hat{q}(\tilde{v}|p_1) = \mathbb{P}\{v = \tilde{v}|p_1\} \in [0, 1] \quad \forall \tilde{v} \in V$$

the seller's posterior belief that the buyer type is  $\tilde{v}$ , conditional on the event that the buyer rejects  $p_1$  in the first round but still continues the negotiation in the second round.

We will focus on perfect Bayesian equilibrium, which will be referred to as simply "equilibrium" hereafter. Holding all other parameters fixed, let  $\mathcal{E}(\epsilon)$  denote the set of all equilibria for the case where the probability of Nature's intervention at the end of the first round is given by  $\epsilon$ . An equilibrium in  $\mathcal{E}(\epsilon)$  is generically denoted by  $(\sigma, \tau, \hat{q})$ . We will say that there is an essentially unique equilibrium if the same outcome occurs along the path of all equilibria in  $\mathcal{E}(\epsilon)$ .

## 2.2 Equilibrium Characterization

In this subsection, we focus on the case where  $\epsilon = 0$ . The analysis of the case that  $\epsilon > 0$  is essentially identical and can be found in Appendix A. Fix any equilibrium, and suppose that the seller offers  $p_1$  in the first round, where  $p_1$  may or may not be in the support of the seller's equilibrium mixed strategy. In addition, suppose that some buyer type chooses to delay in the first round in response to  $p_1$ . Then, the seller's posterior belief at the beginning of the second round (regarding the buyer's type) is well-defined, and let  $\underline{v}(p_1)$  denote the lowest buyer type in the support of the seller's posterior belief. In addition, let  $\underline{p}_2(p_1)$  denote the lowest  $p_2$  that the seller's equilibrium strategy will ever choose in the second round.

The seller will never offer strictly lower than  $\bar{p}(\underline{v}(p_1))$  in the second round,<sup>5</sup> and thus,

$$\delta[\underline{v}(p_1) - \underline{p}_2(p_1)] \leq \delta[\underline{v}(p_1) - \bar{p}(\underline{v}(p_1))] \leq \delta[\underbrace{\underline{v}(p_1) - u(\underline{v}(p_1))}_{=w} + \Delta] < [\delta + (1 - \delta)(1 - \epsilon)]w = w,$$

where the last two inequalities follows the definition of  $\Delta$  and Assumption (A2), respectively. Thus, the buyer type  $\underline{v}(p_1)$  could get better off by exercising the outside option in the first round rather than delaying, contradictory with the supposition that this buyer type belongs to the support for the seller's posterior belief.

The argument in the last paragraphs reveals that no delay could occur in the first round in response to any price offered by the seller. Thus, if the seller offers  $p_1$  in the first round, the buyer accepts it if and only if  $u(v) > p_1$  or exercises the outside option immediately. The seller's equilibrium offer in the first round must solve the following maximization problem:

$$\max_{p_1 \geq 0} \sum_{v \in V: u(v) \geq p_1} p_1 \cdot q(v). \quad (2.1)$$

The next proposition summarizes the discussion so far and shows that the main theoretical prediction of BP also holds for our two-round bargaining game in the essentially unique equilibrium. The proof can be found in Appendix A.

**Proposition 1.** *Fix all parameters other than  $\epsilon$ , and suppose that the maximization problem (2.1) admits a unique solution. Then, there is a cutoff  $\bar{\epsilon} > 0$  such that there is an essentially unique equilibrium whenever  $\epsilon \in (0, \bar{\epsilon})$ . Furthermore, in the essentially unique equilibrium:*

- (i) *The seller's equilibrium offer in the first round solves the maximization problem (2.1).*
- (ii) *Suppose  $\epsilon = 0$ . Both on and off the equilibrium path, the buyer accepts  $p_1$  (i.e.,  $\sigma_v(A|p_1) = 1$ )*

<sup>5</sup>By definition of  $\bar{p}(\cdot)$  and  $\underline{v}(p_1)$ ,  $v - \bar{p}(\underline{v}(p_1)) \geq \underline{v}(p_1) - \bar{p}(\underline{v}(p_1)) > w$  for any  $v$  in the support of the seller's posterior belief. Thus, all buyer types in the support of the seller's posterior belief will accept any  $p_2 \leq \bar{p}(\underline{v}(p_1))$  for sure. Given this, the seller finds any  $p_2 < \bar{p}(\underline{v}(p_1))$  strictly suboptimal.

if and only if  $u(v) > p_1$ ; otherwise, the buyer exercises the outside option immediately (i.e.,  $\sigma_v(O|p_1) = 1$ ).

(iii) For any  $(\epsilon^n)_{n=1}^\infty \in (0, \bar{\epsilon})^\mathbb{N}$  and  $((\sigma^n, \tau^n, \hat{q}^n))_{n=1}^\infty \in \prod_{n=1}^\infty \mathcal{E}(\epsilon^n)$  such that  $\epsilon^n \downarrow 0$ ,

$$\begin{aligned} \lim_{n \rightarrow \infty} \sigma_v^n(A|p_1) = 1 &\iff u(v) > p_1, \\ \lim_{n \rightarrow \infty} \sigma_v^n(O|p_1) = 1 &\iff u(v) < p_1. \end{aligned}$$

On top of the essentially unique equilibrium that renders us a clear benchmark for experiments, there are at least three points worth discussing. First, the buyer's action must be either accepting the seller's offer  $p_1$  or exercising the outside option with probability approaching 1 as  $\epsilon \rightarrow 0$ . This holds even when  $p_1$  is not the equilibrium offer. Second, for this equilibrium to arise, the seller must internalize that lower-type buyers' exercise of the outside option, which results in the positive selection of the remaining demand pool. Third, knowing that any buyers with  $u(v) > p_1$  will accept the price offer, the seller should solve the profit maximization problem (2.1).

Note that the maximization problem (2.1) yields the theoretically highest profit that the seller could ever earn in this bargaining environment, which could be attainable if the seller could fully commit to the optimal price path before the bargaining game begins. With the buyer's outside option, the seller could achieve this theoretical benchmark in the essentially unique equilibrium, even without the full commitment power. This stands in contrast with the case that the buyer has no outside option, in which case the seller's equilibrium profit in a two-round bargaining environment is typically strictly lower than one from committing to the optimal price path in advance.<sup>6</sup>

This sharp contrast arises because a delay in the negotiation signals different types of information in the two cases. Without an outside option, a delay always indicates negative selection in the demand pool, which in turn induces a more pessimistic belief (compared to the prior) of the seller in the second round. This negative selection ultimately leads to a lower price in the second round, thereby eroding the seller's bargaining power in the first round. When the buyer has an outside option as in our model, however, a delay could signify positive selection in the demand pool because low buyer types have a larger incentive to exercise their outside option earlier than high types. This positive selection allows the seller to be more confident in charging a high price in the second round, which ultimately provides the seller with stronger bargaining power in the first round.<sup>7</sup>

Let us make a couple of more remarks before we close this section. First, note that the equilibrium outcome is qualitatively the same in both cases  $\epsilon = 0$  and  $\epsilon > 0$  (provided that  $\epsilon$  is small), and thus, Nature's intervention does not have much theoretical implications. However, the possibility of

<sup>6</sup>The analysis of the case without outside option can be found, for example, in [Fudenberg and Tirole \(1991, Section 10.2\)](#).

<sup>7</sup>In Section 3, we will delve further into how positive selection arises and how it affects the dynamics of bargaining.



Nature’s intervention still facilitates the analysis of data from the second round. Without Nature’s intervention, any observations made in the second round would be considered off the equilibrium path and thus cannot be theoretically pinned down. The possibility of Nature’s intervention for the case  $\epsilon > 0$  allows us to obtain testable theoretical predictions for the seller’s posterior beliefs off the equilibrium path.

Finally, the results have an important policy implication, especially for the market design and regulatory policy in various markets. If the market designer’s goal is to protect consumer surplus, then BP’s result suggests that when there are any outside options, the seller’s surplus, not the buyer’s, is maximized. However, this policy implication seems contrary to the conventional wisdom that access to outside options usually enhances consumer surplus. Such a gap between our conventional wisdom and economic theory motivates our approach of using controlled laboratory experiments.

### 3 Experimental Design and Hypotheses

#### 3.1 Experimental Design

As considered in Section 2, we consider a two-round ( $t = 1, 2$ ) bargaining game between a seller and a buyer with one-sided private information in our experiment. The seller has an indivisible good for sale. The buyer’s value of the good  $v$  is drawn from the finite support  $\{v_L, v_M, v_H\}$  prior to the negotiation, and it is private information of the buyer. We call  $v_L$ ,  $v_M$ , and  $v_H$  the low type, middle type, and high type, respectively. The value of the outside option is given as  $w$ , which is commonly known and independent of the realization of  $v \in V$ . We use the following parameter values:  $v_H = 500$ ,  $v_L = 70$ ,  $v_M \in \{90, 240, 420\}$ , and  $w = 50$ . The prior belief is given by  $\mathbb{P}\{v = v_L\} = \mathbb{P}\{v = v_M\} = \mathbb{P}\{v = v_H\} = \frac{1}{3}$ .

In each round  $t = 1, 2$ , the seller offers a price among

$$\mathcal{P} = \{v_L - w - \Delta, v_M - w - \Delta, v_H - w - \Delta\},$$

where we set  $\Delta = 10$  to avoid tie-breaking situations. For notational simplicity, we denote  $v_i - w - \Delta$  by  $p_i$ ,  $i \in \{H, M, L\}$ , so  $\mathcal{P} = \{p_L, p_M, p_H\}$ . With these three possible price offers, a high-type buyer is strictly better off by accepting any of them than taking the outside option, a middle-type buyer is strictly better off by accepting  $p_L$  or  $p_M$ , and a low-type buyer is strictly better off by accepting  $p_L$ . Once offered, the buyer decides whether to accept or reject the price offer or take the outside option. To facilitate bargaining agreement earlier, we introduce the random termination of a match (Roth and Murnighan, 1978) with a fixed continuation probability of  $\delta = 0.8$ .<sup>8</sup> When the buyer

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<sup>8</sup>The continuation probability does not matter much as the theoretical predictions are identical under a broad range of  $\delta$ , but it needs to be strictly smaller than 1. See Section 3.2.

accepts  $p_1$  in round 1, the negotiation ends, and the seller and the buyer receive the respective payoffs of  $p_1$  and  $v - p_1$ . When the buyer rejects  $p_1$ , the negotiation proceeds to the next round with probability  $\delta$ . In case of termination, both the seller and the buyer receive a payoff of zero. When continued, the seller is asked to report her belief about the matched buyer’s type and offer the second round price. When the buyer decides to pursue the outside option, the negotiation ends, and the seller and the buyer receive the respective payoffs of 0 and  $w$ . The experiment consists of one practice match<sup>9</sup> and ten payment-relevant matches, and participants are reshuffled to form new pairs after each match so that there are no strategic dynamics between matches. The participants’ roles were fixed across matches.

Table 1: Experimental Design

<i>M90</i>	<i>M240</i>	<i>M420</i>
$v \in \{70, 90, 500\}$	$v \in \{70, 240, 500\}$	$v \in \{70, 420, 500\}$
* Each participant has ten newly paired matches.		
* Continuation probability to the next round is 0.8.		
* Buyer’s value $v$ is uniformly drawn from $\{70, v_M, 500\}$ .		

Our experimental design is summarized in Table 1. Three treatment conditions differ in the value of the middle type, so we call *M90* if the middle type’s value is 90. *M240* and *M420* are called accordingly. Our experiment was conducted by oTree (Chen et al., 2016) at the HKUST. A total of 294 subjects were recruited from the graduate and undergraduate population of the university. We had 5 sessions per treatment. Each session consisted of 16 to 22 participants, and we had 106, 100, and 88 participants in treatments *M90*, *M240*, and *M420*, respectively. In all sessions, subjects participated in ten matches of the bargaining game under one treatment condition. Sample experimental instructions can be found in Appendix B.

### 3.2 Justification of Parameter Values

In choosing  $\Delta$  and other parameter values, we make sure rejecting the price offer in the first round is strictly dominated by taking the outside option:

$$\delta[v - (u(v) - \Delta)] = \delta(w + \Delta) < (1 - \epsilon)w + \delta\epsilon w \quad \forall v \iff \delta\Delta < (1 - \epsilon)(1 - \delta)w. \quad (3.1)$$

From equation (3.1), we restrict  $\delta$  to be strictly smaller than 1, and  $\Delta$  to be sufficiently small. Given  $w = 50$ ,  $\delta = 0.8$ , and  $\epsilon \approx 0$ ,  $\Delta < 12.5$ . So we set  $\Delta = 10$ . Note that our choice of parameters here is also consistent with the assumption (A2) in the last section.

<sup>9</sup>We exclude this practice match from our analysis, as we clarified that the purpose of the practice match is to familiarize themselves with the experiment, and their performance in that practice round is not relevant for payment. Including this practice match data does not change any results in a meaningful way.

Also, given these parameter values, we make sure that offering a price that is too low should not be a profit maximizing strategy for the seller. Specifically, we require at least one of the following two conditions to hold:

$$p_L \cdot [q(v_H) + q(v_M) + q(v_L)] = p_L < p_H \cdot q(v_H) \quad (3.2)$$

$$p_L \cdot [q(v_H) + q(v_M) + q(v_L)] = p_L < p_M \cdot [q(v_H) + q(v_M)] \quad (3.3)$$

Equation (3.2) means that selling the good only to the high type with the high price is better than selling the good to every buyer with the low price, and equation (3.3) means that selling the good to the high and middle types with the middle price is better off than selling the good to every buyer with the low price. If those two conditions do not hold, we cannot observe meaningful dynamics between buyers and sellers; The sellers should offer the low price, and all buyers should accept the offer. At least one of the conditions hold in our parameter values,  $v_L = 70$ ,  $v_H = 500$ ,  $w = 50$ , and  $q(v_H) = q(v_M) = q(v_L) = \frac{1}{3}$ , with leaving a room for varying the value for the median type.

Let  $p^*$  denote the solution for the following optimization problem (the seller's static profit maximization problem):

$$\max_{p \in \mathcal{P}} \sum_{v: u(v) \geq p} p \cdot q(v). \quad (3.4)$$

Then,

$$p^* = \begin{cases} p_H = 440 & \text{if } v_M = 90 \text{ or } 240. \\ p_M = 360 & \text{if } v_M = 420. \end{cases} \quad (3.5)$$

We vary the value of the middle type so that the equilibrium price offer varies accordingly. In words, if the middle type value is not too high, it is optimal to sell the good only to the high-type buyers, and otherwise, it is optimal to sell the good to the middle- and high-type buyers.

Lastly, we set the value of  $\epsilon = 0.001$  so that the introduction of  $\epsilon$  does not practically change the equilibrium.

### 3.3 Theoretical Predictions and Hypotheses

Table 2 summarizes the theoretical predictions by treatment. For example, in  $M240$ , the seller offers 440 in round 1, the high-type buyer accepts the offers, and the other-type buyer takes the outside option. Delay due to rejection of the first-round price offer should not be observed. If ever moved to Round 2 because of  $\epsilon$ , they repeat the same strategy. Note that the theoretical predictions for  $M90$  and  $M240$  are identical, although  $v_M$  varies. On top of examining treatment effects between  $M240$  and  $M420$ , we intend to check no treatment effects between  $M90$  and  $M240$ . This enables us to enunciate whether the treatment effect is consistent with the theory.

Our first check must be to examine the consistency between our experimental data and the

	M90	M240	M420
*Seller offers	440	440	360
*Buyer with $v_L$ plays	O	O	O
*Buyer with $v_M$ plays	O	O	A
*Buyer with $v_H$ plays	A	A	A
Expected Payoffs of Seller	146.67	146.67	240
(Ex ante) Expected Payoffs of Buyer	53.33	53.33	83.33

O: takes the outside option. A: accepts the offer

\*: If moved to Round 2, repeat the same procedure.

Table 2: Summary of Theoretical Predictions

theoretical predictions in Table 2. Since the theoretical predictions are built upon many layers of reasoning, we dissect our theoretical predictions so that we can delve further into examining how experimental evidence is consistent with different levels of reasoning. In what follows, to simplify notation, we will write  $\sigma_H$ ,  $\sigma_M$ , and  $\sigma_L$  as the shorthand for  $\sigma_{v_H}$ ,  $\sigma_{v_M}$ , and  $\sigma_{v_L}$ , respectively.

**First-Order Positive Selection** The “minimal” rationality we require is that the low type should never choose to delay; this is the case as long as equation (3.1) holds. Note that this is particularly convincing once we restrict the seller’s offer in the second round to  $p_2 \in \mathcal{P} = \{p_L, p_M, p_H\}$ . Summing up, the lowest buyer type does not choose to reject in the first round. That is,  $\sigma_L(D|p_1) = 0$  for any  $p_1 \in \mathcal{P}$ .

Given  $\sigma_L(D|p_1) = 0$ , as long as the seller believes that the buyer is “minimally” rational, the seller must put zero probability to the event that the lowest buyer type chooses to delay in the first round (although Nature would force the buyer to delay with probability  $\epsilon > 0$ , despite that the buyer does not choose to delay). The seller’s posterior belief about the low-type buyer should be

$$\begin{aligned} \hat{q}(v_L|p_1) &= \frac{\epsilon q(v_L)}{\epsilon + (1 - \epsilon)[q(v_L)\sigma_L(D|p_1) + q(v_M)\sigma_M(D|p_1) + q(v_H)\sigma_H(D|p_1)]} \\ &= \frac{\epsilon}{3\epsilon + (1 - \epsilon)[\sigma_M(D|p_1) + \sigma_H(D|p_1)]}, \end{aligned} \quad (3.6)$$

where the last equality follows  $\sigma_L(D|p_1) = 0$  and  $q(v) = 1/3$  for all  $v$ .

Without any further restrictions to  $\sigma_M(D|p_1) \in [0, 1]$  and  $\sigma_H(D|p_1) \in [0, 1]$ , we can only posit the following range of  $\hat{q}(v_L|p_1)$ :

$$0 \approx \frac{\epsilon}{\epsilon + 2} = \frac{\epsilon}{3\epsilon + (1 - \epsilon)[1 + 1]} \leq \hat{q}(v_L|p_1) \leq \frac{\epsilon}{3\epsilon + (1 - \epsilon)[0 + 0]} = \frac{1}{3},$$

where the first weak inequality holds as equality if and only if  $\sigma_M(D|p_1) = \sigma_H(D|p_1) = 1$ , and the second weak inequality holds as equality if and only if  $\sigma_M(D|p_1) = \sigma_H(D|p_1) = 0$ . Furthermore,

both

$$\hat{q}(v_M|p_1) = \frac{q(v_M)[\epsilon + (1-\epsilon)\sigma_M(D|p_1)]}{\epsilon + (1-\epsilon)[q(v_M)\sigma_M(D|p_1) + q(v_H)\sigma_H(D|p_1)]} = \frac{\epsilon + (1-\epsilon)\sigma_M(D|p_1)}{3\epsilon + (1-\epsilon)[\sigma_M(D|p_1) + \sigma_H(D|p_1)]}$$

and

$$\hat{q}(v_H|p_1) = \frac{q(v_H)[\epsilon + (1-\epsilon)\sigma_H(D|p_1)]}{\epsilon + (1-\epsilon)[q(v_M)\sigma_M(D|p_1) + q(v_H)\sigma_H(D|p_1)]} = \frac{\epsilon + (1-\epsilon)\sigma_H(D|p_1)}{3\epsilon + (1-\epsilon)[\sigma_M(D|p_1) + \sigma_H(D|p_1)]}$$

are always weakly larger than  $\hat{q}(v_L|p_1)$ , where

$$0 \approx \frac{\epsilon}{3\epsilon + (1-\epsilon)} = \frac{\epsilon + (1-\epsilon) \cdot 0}{3\epsilon + (1-\epsilon)[0+1]} \leq \hat{q}(v_M|p_1) \leq \frac{\epsilon + (1-\epsilon) \cdot 1}{3\epsilon + (1-\epsilon)[1+0]} = \frac{1}{3\epsilon + (1-\epsilon)} \approx 1,$$

$$0 \approx \frac{\epsilon}{3\epsilon + (1-\epsilon)} = \frac{\epsilon + (1-\epsilon) \cdot 0}{3\epsilon + (1-\epsilon)[1+0]} \leq \hat{q}(v_H|p_1) \leq \frac{\epsilon + (1-\epsilon) \cdot 1}{3\epsilon + (1-\epsilon)[1+0]} = \frac{1}{3\epsilon + (1-\epsilon)} \approx 1.$$

This thought process on the first-order positive selection leads us to the following hypothesis.

**Hypothesis 1** (First-order positive selection). *No low-type buyers choose to delay ( $\sigma_L(D|p_1) = 0$ ), and thus,  $\hat{q}(v_L|p_1) \leq q(v_L) = 1/3$  for any  $p_1 \in \mathcal{P}$ . If ever moved to the second round, the posterior belief that a buyer is a high/middle type is weakly greater than the posterior belief that a buyer is a low type, for any price offer in round 1. That is,  $\hat{q}(v_L|p_1) \leq \min\{\hat{q}(v_M|p_1), \hat{q}(v_H|p_1)\}$  for any  $p_1 \in \mathcal{P}$ .*

**Second-Order Positive Selection** Given Hypothesis 1, a rational seller will never offer  $p_2 = p_L$  in the second round. To see this, note that the seller's expected payoff from  $p_2 = p_L$  is  $p_L \cdot [\hat{q}(v_L) + \hat{q}(v_M) + \hat{q}(v_H)] = p_L$ . On the other hand, the seller's expected payoff from  $p_2 = p_M$  is  $p_M \cdot [1 - \hat{q}(v_L)]$ . The difference between the two payoffs is

$$p_M \cdot [1 - \hat{q}(v_L)] - p_L \geq p_M \cdot [q(v_M) + q(v_L)] - p_L > 0,$$

where the weak inequality follows Hypothesis 1 ( $1 - \hat{q}(v_L) \geq 2/3 = q(v_H) + q(v_M)$ ) and the strict inequality follows the maintained condition in equation (3.3). Then, by following a similar reasoning for  $\sigma_L(D|p_1) = 0$ , the middle type also finds it strictly suboptimal to reject the price offer in round 1 and delay the negotiation to round 2. Thus,  $\sigma_M(D|p_1) = 0$  and

$$\begin{aligned} \hat{q}(v_M|p_1) &= \frac{\epsilon q(v_M)}{\epsilon + (1-\epsilon)[q(v_L)\sigma_L(D|p_1) + q(v_M)\sigma_M(D|p_1) + q(v_H)\sigma_H(D|p_1)]} \\ &= \frac{\epsilon q(v_M)}{\epsilon + (1-\epsilon)q(v_H)\sigma_H(D|p_1)} = \frac{\epsilon}{3\epsilon + (1-\epsilon)\sigma_H(D|p_1)}, \end{aligned} \quad (3.7)$$

where the last equality follows  $\sigma_L(D|p_1) = \sigma_M(D|p_1) = 0$ .

With  $\sigma_M(D|p_1) = \sigma_L(D|p_1) = 0$ ,

$$\hat{q}(v_L|p_1) = \hat{q}(v_M|p_1).$$

Additionally, without any further restriction to  $\sigma_H(D|p_1)$ ,

$$0 \approx \frac{\epsilon}{3\epsilon + (1-\epsilon) \cdot 1} \leq \hat{q}(v_L|p_1) = \hat{q}(v_M|p_1) \leq \frac{\epsilon}{3\epsilon + (1-\epsilon) \cdot 0} = \frac{1}{3}$$

and

$$\frac{1}{3} = \frac{\epsilon + (1-\epsilon) \cdot 0}{3\epsilon + (1-\epsilon) \cdot 0} \leq \hat{q}(v_H|p_1) = \frac{\epsilon + (1-\epsilon)\sigma_H(D|p_1)}{3\epsilon + (1-\epsilon)\sigma_H(D|p_1)} \leq \frac{\epsilon + (1-\epsilon) \cdot 1}{3\epsilon + (1-\epsilon) \cdot 1} \approx 1.$$

In summary, from the second-order thought process of positive selection, we know  $\sigma_L(D|p_1) = \sigma_M(D|p_1) = 0$  and  $\hat{q}(v_L|p_1) = \hat{q}(v_M|p_1) \leq \frac{1}{3} = q(v_H) \leq \hat{q}(v_H)$ . This leads us to our second hypothesis.

**Hypothesis 2** (Second-order positive selection). *No middle-type buyers choose to delay ( $\sigma_M(D|p_1) = 0$ ), and thus,  $\hat{q}(v_M) \leq q(v_M) = 1/3$  for any  $p_1 \in \mathcal{P}$ . If ever moved to the second round, the posterior belief that a buyer is a high type is weakly greater than the posterior belief that a buyer is a low/middle type, for any price offer in round 1. That is,  $\hat{q}(v_L|p_1) = \hat{q}(v_M|p_1) \leq \hat{q}(v_H|p_1)$  for any  $p_1 \in \mathcal{P}$ .*

**Third-order Positive Selection** For the third-order reasoning, we separate cases based on the static monopolistic prices. First, consider the case  $v_M = 90$  or  $240$  where  $p^* = u(v_H) - \Delta = 440$ . The first- and second-order positive selections jointly imply the posterior belief  $\hat{q}$  over the finite support  $\{v_L, v_M, v_H\}$  first-order stochastically dominates the prior belief  $q$ . Since the seller solves the static profit maximization problem, the seller's price offer in the second round must be weakly larger than  $p^*$ . Given there are only three price options,  $p_2$  must be binding to  $u(v_H) - \Delta$ . This implies that the high-type buyer also has no reason to choose delay (i.e.,  $\sigma_H(D|p_1) = 0$ ). Thus, by the reasoning similar to the first- and second-order positive selections,

$$\hat{q}(v_H|p_1) = \frac{\epsilon}{3\epsilon + (1-\epsilon)\sigma_H(D|p_1)} = \frac{\epsilon}{3\epsilon + (1-\epsilon) \cdot 0} = \frac{1}{3} = q(v_H). \quad (3.8)$$

Substituting  $\sigma_H(D|p_1) = 0$  in (3.6) and (3.7), we also obtain  $\hat{q}(v_M) = \hat{q}(v_L) = 1/3$ . We obtain the following hypothesis.

**Hypothesis 3.A** (Third-order positive selection). *Suppose  $v_M = 90$  or  $240$  so that  $p^* = u(v_H) - \Delta$ . No high-type buyers choose to delay ( $\sigma_H(D|p_1) = 0$ ). If ever moved to the second round, the posterior belief is the same as prior for any type and any price offer in round 1. In equilibrium, the seller offers  $p^* = u(v_H) - \Delta$  in round 1, the high-type buyer accepts  $p^*$ , and other types exercise the outside option.*

Next, consider the case  $v_M = 420$  where  $p^* = p_M = 360$ . There are two subcases. The first subcase regards when the seller offers  $p_1 \in \{p_L, p_M\}$  in the first round. Then, by the same reasoning in the previous case ( $v_M = 90$  or  $240$ ), the posterior belief  $\hat{q}$  first-order stochastically dominates the prior belief  $q$ , so the seller's profit-maximizing price offer in the second round must be higher or equal to the price offer in the first round. With knowing this, all buyer types would not choose to delay in the first round; i.e.,  $\sigma_L(D|p_1) = \sigma_M(D|p_1) = \sigma_H(D|p_1) = 0$  if  $p_1 \in \{p_L, p_M\}$ .

The second subcase regards when the seller offers  $p_1 = p_H$  in the first round. In this situation, the high-type buyer must randomize between accepting and rejecting the price offer. (Exercising the outside option is strictly dominated by accepting  $p_1 = p_H$ .) To see why, suppose that the high type chooses to delay with probability 1. Then, given the first- and second-order positive selections, the seller who believes  $\hat{q}(v_H|p_1) \approx 1$  in the second round will offer  $p_2 = p_H$ . However, the high type would find it suboptimal to delay in the first place, especially when  $\delta < 1$ . Suppose, on the contrary, that the high type chooses to accept  $p_1 = p_H$  with probability 1 and obtains  $w + \Delta = 60$  as a final payoff. Then the seller who believes  $\hat{q}(\cdot) = q(\cdot)$  would offer  $p_2 = p^* = p_M = 360$  in the second round. By rejecting the first price offer to accept the second offer, the buyer expects to earn  $\delta(v_H - p_2) = \delta(500 - 360) = 112$ , which is greater than the payoff of accepting  $p_1$ . Therefore, the high-type buyer must choose to delay with some probability  $\sigma_H(D|p_1 = p_H)$ . To justify the high-type buyer's randomization, the seller also must randomize between offering  $p_2 = p_M$  and  $p_2 = p_H$  in the second round. The seller is willing to randomize only if

$$p_H \cdot \hat{q}(v_H|p_1 = p_H) = p_M \cdot [\hat{q}(v_H|p_1 = p_H) + \hat{q}(v_M|p_1 = p_H)] \iff \frac{\hat{q}(v_H|p_1 = p_H)}{\hat{q}(v_M|p_1 = p_H)} = \frac{p_M}{p_H - p_M},$$

which is the case if and only if

$$\sigma_H(D|p_1 = p_H) = \frac{\epsilon}{1 - \epsilon} \frac{p_M - (p_H - p_M)}{p_H - p_M}.$$

Note that  $\sigma_H(D|p_1 = p_H) \rightarrow 0$  as  $\epsilon \rightarrow 0$ . Thus, with small  $\epsilon \approx 0$ , the probability of a delay occurring in the first round is very small. With our choices  $\epsilon = 0.001$ ,  $\Delta = 10$ ,  $v_H = 500$ , and  $v_M = 420$ ,

$$\sigma_H(D|p_1 = p_H) = \frac{0.001}{0.999} \frac{280}{80} \approx 0.0035 \quad \text{and} \quad \frac{\hat{q}(v_H|p_1 = p_H)}{\hat{q}(v_L|p_1 = p_H)} = \frac{\hat{q}(v_H|p_1 = p_H)}{\hat{q}(v_M|p_1 = p_H)} = \frac{p_M}{p_H - p_M} = \frac{360}{80} = \frac{9}{2}.$$

Combining  $\frac{\hat{q}(v_H|p_1 = p_H)}{\hat{q}(v_L|p_1 = p_H)} = \frac{9}{2}$  with  $\hat{q}(v_L|p_1 = p_H) = \hat{q}(v_M|p_1 = p_H)$  and  $\hat{q}(v_L|p_1) + \hat{q}(v_M|p_1) + \hat{q}(v_H|p_1) = 1$ , we draw  $\hat{q}(v_L|p_1 = p_H) = \hat{q}(v_M|p_1 = p_H) = \frac{2}{13}$  and  $\hat{q}(v_H|p_1 = p_H) = \frac{9}{13}$ . Summing up, we obtain the following hypothesis.

**Hypothesis 3.B** (Third-order positive selection). *Suppose  $v_M = 420$  so that  $p^* = p_M$ . (i) If  $p_1 \in \{p_L, p_M\}$ , then no high-type buyers choose to delay ( $\sigma_H(D|p_1) = 0$ ). If ever moved to the second round, the posterior belief is the same as the prior. (ii) If  $p_1 = p_H$ , then high-type buyers*

rarely choose to delay ( $\sigma_H(D|p_1) \approx 0$ ). If ever moved to the second round, the posterior belief that a buyer is a high type is nearly 70% (9/13). In equilibrium, the seller offers  $p_1 = p^* = p_M$ , and thus, no delay occurs.

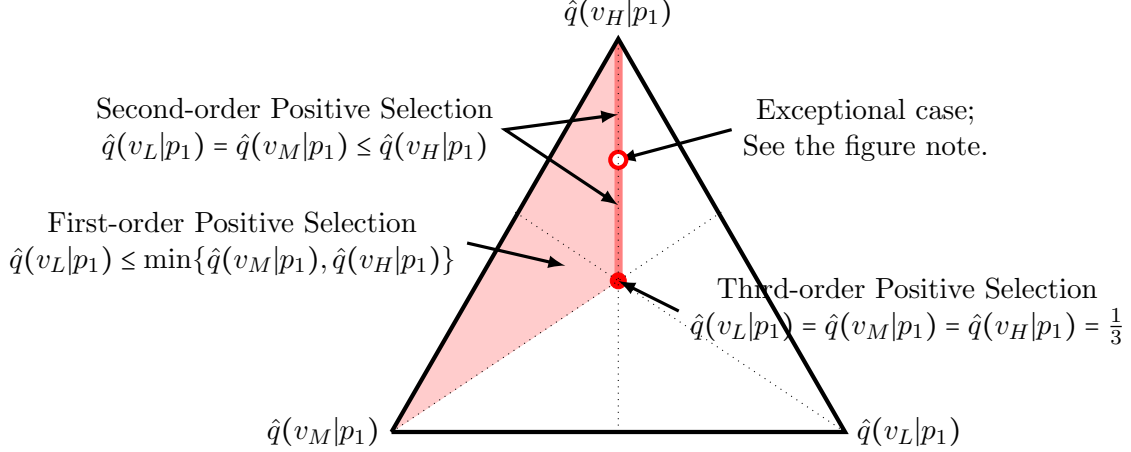


Figure 1: Positive Selections and Posterior Beliefs

The unit simplex represents all possible posterior beliefs after the first-round rejection. The shaded area and the vertical line respectively represent the set of posterior beliefs admissible under the first- and second-order positive selection. The filled circle at the center represents the posterior belief under the third-order positive selection, except for the case where  $p_1 = p_H$  in *M420*. The hollow circle near the top vertex represents the posterior belief under the third-order positive selection for the exceptional case.

Figure 1 illustrates the ranges of rationalizable posterior beliefs in the first-, second-, and third-order positive selections. Any posterior belief  $(\hat{q}(v_L|p_1), \hat{q}(v_M|p_1), \hat{q}(v_H|p_1))$  can be described as a point on the unit simplex. The first-order positive selection implies that  $\hat{q}(v_L|p_1)$  is smaller than both  $\hat{q}(v_M|p_1)$  and  $\hat{q}(v_H|p_1)$ , which is represented in the top-left shaded triangle. The second-order positive selection restricts the rationalizable posterior beliefs to a line such that  $\hat{q}(v_L|p_1) = \hat{q}(v_M|p_1) \leq \hat{q}(v_H|p_1)$ , which is represented in the vertical line from the top vertex to the center of the unit simplex. Furthermore, except for the case of  $v_M = 420$  and  $p_1 = p_H$ , the third-order positive selection restricts the posterior beliefs to a point at  $\hat{q}(v_L|p_1) = \hat{q}(v_M|p_1) = \hat{q}(v_H|p_1)$ , the red dot at the center of the figure. The posterior belief under the third-order positive selection for the exceptional case of  $p_1 = p_H$  and  $v_M = 420$  is depicted as the hollow circle near the top vertex.

It is worth noting that the earlier hypothesis *neests* the later one. If our experimental data do not support the theoretical predictions, these three hypotheses can provide us with clear identification from which order subjects fail to follow the reasoning of positive selections.

**The Seller's Price Offers** Suppose that the buyer has rejected  $p_1$  in the first round. Given any posterior beliefs  $\hat{q}(\cdot)$ , the seller's equilibrium offer  $p_2$  must solve the following maximization



problem:

$$\max_{p_2 \in \mathcal{P}} \sum_{v: u(v) \geq p_1} p_2 \cdot \hat{q}(v|p_1) \quad \forall p_1 \in \mathcal{P}. \quad (3.9)$$

Since we solicit the seller to report posterior beliefs, we can test the following hypothesis.

**Hypothesis 4.** *Given the seller’s reported posterior belief  $\hat{q}(\cdot)$ , the price offered in the second round maximizes the seller’s expected payoff.*

By testing Hypotheses 1–4 together, we can check the validity of BP’s theoretical prediction and also identify in which part drops the ball. If, for example, Hypotheses 1–3.B are supported but Hypothesis 4 is rejected, then it is the seller’s failure to maximize the expected payoff given his self-reported posterior beliefs. On the other hand, if Hypothesis 4 is supported but any of Hypotheses 2, 3.A, and 3.B are not, then it is the seller’s failure of higher-order positive selection reasoning. If we reject Hypothesis 1, then the inconsistency between the theoretical prediction and experimental evidence would be mostly due to the failure of Bayesian updating.

In the case of  $v_M \in \{240, 90\}$ ,  $\sigma_H(D|p_1) = \sigma_M(D|p_1) = \sigma_L(D|p_1) = 0$  for any  $p_1 \in \mathcal{P}$ . This is the case only if the buyer expects that delaying the negotiation in the first round is not worth:

$$\mathbb{E}[\delta \max\{v - p_2, w\}] \leq \max\{w, v - p_1\} \quad \forall v, p_1, \quad (3.10)$$

where the expectation on the left-hand side of the inequality is taken with respect to the buyer’s expectation about the seller’s offer  $p_2$  in the second round. If this is not the case, then some buyer type in  $\{v_H, v_M, v_L\}$  will find it optimal to delay in the first round, contradictory with the prediction of positive selection. Since we have three types and three price offers available, we expand (3.10) in Table 3 to show in which case the inequality can be violated. When either  $v = v_L$  or  $p_1 = p_L$ , the inequality always holds regardless of the buyer’s subjective beliefs. When  $p_1 = p_M$  and  $v \in \{v_M, v_H\}$ , the inequality can be violated if the buyer believes  $p_2 = p_L$  is highly likely. When  $p_1 = p_H$  and  $v \in \{v_M, v_H\}$ , the inequality can be violated if the buyer believes  $p_2 \leq p_M$  is highly likely. Since the violation condition for  $p_M$  (shaded in blue) is more stringent than that for  $p_H$  (shaded in red), the inequality holds for any  $v$  and any  $p_1$  unless buyers expect high price offers in the first round would drop to lower price offers in the second round. In other words, if the experimental data shows some “delays,” then it is most likely in the  $v - p_1$  pair shaded in red, somewhat likely in the pair shaded in blue, and not likely in other pairs. This leads to the following hypothesis.

**Hypothesis 5.** *Buyers with  $v_M$  or  $v_H$  are more likely to delay in response to  $p_H$ , somewhat likely to delay in response to  $p_M$ , and not likely to delay in response to  $p_L$ . Buyers with  $v_L$  are not likely to delay in response to any price offer in round 1.*

Testing Hypothesis 5 enables us to gather insights into whether the buyer’s decision to delay

$\mathbb{E}[\delta \max\{v - p_2, w\}] \leq \max\{w, v - p_1\}$ ?			
$v \setminus p_1$	$p_L$	$p_M$	$p_H$
$v_L$	always hold	always hold	always hold
$v_M$	always hold	can be violated if $p_2 = p_L$	can be violated if $p_2 \leq p_M$
$v_H$	always hold	is expected too much	is expected too much

Table 3: Validity of not expecting “too low price” in Round 2

is influenced by their misspecified expectation of the second-round price offer. If the buyer’s expectation violates (3.10), it implies that the buyer anticipates the second-round price offer to be sufficiently low, justifying the decision to delay. If our observations are consistent with Hypothesis 5, it would indicate that buyers’ decisions are rational given their misspecified beliefs. Conversely, a lack of consistency might suggest that their decisions are driven by some other fundamental reasoning failure.

Finally, in the first round, the seller’s offer must maximize the seller’s static profit:

$$\max_{p_1 \in \mathcal{P}} \sum_{v \in V: u(v) \geq p_1} p_1 \cdot q(v). \quad (3.11)$$

In other words, the seller offers  $p_1 = p^*$  in equilibrium. No delay occurs in equilibrium.

**Hypothesis 6.** *The observed  $p_1$  solves the maximization problem (3.11). No delay occurs.*

## 4 Results

In this section, we first report the summary of experimental findings and then present each part of experimental findings in the corresponding order of our hypotheses. Throughout this section, “Rejection of  $p_1$ ” is a shorthand for the event that the buyer chooses to delay in response to the seller’s offer  $p_1$ , neither accepting  $p_1$  nor exercising the outside option in the first round.

Table 4 shows the overall experimental findings, juxtaposing theoretical predictions. Except for a few coincidental numbers, there are substantial differences between experimental findings and theoretical predictions. The seller’s equilibrium price offer is the smallest in  $M420$ , but the seller’s average price offer shows the opposite. The seller’s equilibrium price offer in  $M90$  is identical to that in  $M240$ , but the average offer in  $M90$  is significantly larger than that in  $M240$ . The average payoff<sup>10</sup> of the seller is the largest in  $M420$ , but that is significantly smaller than the expected payoff. The buyer’s average payoff in  $M420$  was the smallest, while theory predicts the opposite.

A substantial proportion of rejections is one of the most distinctive inconsistencies we observe

<sup>10</sup>Since one of the ten matches was randomly selected for payment, the average payoff (the average of ten matches) is different from what the experiment participants actually earned.

	<i>M90</i>	<i>M240</i>	<i>M420</i>
Avg.Offer (Theory)	346.25 (440)	295.86 (440)	378.50 (360)
%Reject_ $v_L$ (Theory)	39 (0)	35 (0)	32 (0)
%Reject_ $v_M$ (Theory)	51 (0)	60 (0)	74 (0)
%Reject_ $v_H$ (Theory)	63 (0)	50 (0)	59 (0)
Avg.SellerPayoffs (Theory)	54.43 (146.67)	87.94 (146.67)	165.16 (240)
Avg.BuyerPayoffs (Theory)	111.60 (53.33)	119.20 (53.33)	86.25 (83.33)

Table 4: Summary of Experimental Findings

Avg.Offer is the average of the first-round price offers. %Reject\_  $v_i$  is the percentage that a buyer with type  $v_i$  rejects the price offer. Avg.SellerPayoffs and Avg.BuyerPayoffs are the average payoffs of the sellers and the buyers, respectively. Theoretically predicted values are in parentheses.

from the data. For any buyer type and any price in the first round, the unique equilibrium predicts that every buyer should either accept the first-round price offer or exercise the outside option, so no one should choose to delay. Even when a buyer draws the lowest value so that rejecting does not create a chance for any practically profitable deviations, 39% of the low-type buyers reject in *M90*, 35% and 32% of the low-type buyers reject in *M240* and *M420*, respectively. Middle- and high-type buyers tend to reject more than 50% of the price offer in the first round.

Since we collect the seller’s posterior belief about the matched buyer’s type when the first round ends up with the buyer’s rejection, we could exploit the luxury of 614 observations about the posterior beliefs.<sup>11</sup> Figure 2 shows all reported posterior beliefs. A larger circle indicates more observations in the center of the circle.

The most frequent posterior belief is at  $(\hat{q}(v_L|p_1), \hat{q}(v_M|p_1), \hat{q}(v_H|p_1)) = (0.5, 0.5, 0)$ . The second- and third-most frequent posterior beliefs are  $(0.45, 0.45, 0.1)$  and  $(0.4, 0.4, 0.2)$ , and these three beliefs account 19% (128 out of 614) of the entire observed posterior beliefs. In Figure 2, the posterior beliefs spotted on the top-left shaded area of the unit simplex are the ones that can be rationalized in the first-order positive selection, and about two-thirds of posterior belief reports are outside of the area. This evidence already indicates that the majority of sellers fail at the first-order positive selection reasoning. Note further that the posterior being in the shaded area is a necessary, but not a sufficient, condition supporting the first-order positive selection, so we interpret our observations most favorably toward equilibrium reasoning.

**Result 1.** *51.22% (753 out of 1,470) of the first-round price offers were rejected. Among 614 reported posterior beliefs, 36.64% of them are rationalized in the first-order positive selection.*

It is worth noting that the majority of posterior beliefs seem to reflect the *negative selection*,

<sup>11</sup>The bargaining procedure moves on to the second round with probability  $\delta = 0.8$ , so the 614 posterior beliefs are about 81.5% of the bargaining cases in which the seller’s first-round offer is rejected.

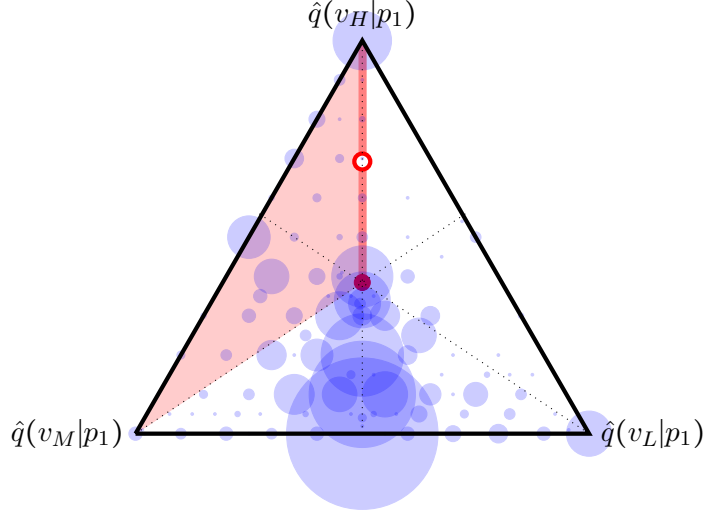


Figure 2: Observed posterior beliefs

believing that the high-type buyers were more likely to accept the first-round price offer so the remaining demand pool consists of more of the middle- and low-type buyers. This reasoning theoretically holds when the outside option does not exist, and it leads to a qualitatively opposite prediction about the equilibrium price dynamics and the seller’s payoff.<sup>12</sup> Also, if the sellers believed that the high-type buyers would accept the price offer and the low-type buyers would take the outside option in the first round, that is, if they applied the positive and the negative selection reasoning simultaneously, then they would have reported that the remaining demand pool mainly consists of the middle type. As illustrated in Figure 2, few posterior beliefs are reported around the vertex of  $\hat{q}(v_M|p_1)$ , indicating that the positive selection reasoning had not been applied. Thus, we claim the sellers’ failure of positive selection reasoning<sup>13</sup> is the main driving force behind the stark discrepancy between the theoretical predictions and our experimental findings.

We further investigate how many reported posterior beliefs pass the second- and third-order positive selections. The condition for the second-order positive selection is  $\hat{q}(v_L|p_1) = \hat{q}(v_M|p_1) \leq \hat{q}(v_H|p_1)$ . We slightly weaken this condition and count the posterior belief reports that satisfy  $\hat{q}(v_L|p_1) \leq \hat{q}(v_H|p_1)$  and  $\hat{q}(v_M|p_1) \leq \hat{q}(v_H|p_1)$ .

**Result 2.** *24.59% of the posterior beliefs (151 out of 614) are rationalized in the second-order positive selection.*

Except for the case of  $p_1 = p_H$  in *M420* where the posterior belief under the third-order positive

<sup>12</sup>This observation is consistent with the findings from our companion paper (Chang et al., 2023) reporting similarities between the bargaining dynamics with and without the outside option.

<sup>13</sup>Some buyers’ behaviors were not rational as well, but even if the sellers would take into account some buyers’ irrational behaviors, the reported posterior beliefs should have been different from what we observe. See the discussions after Result 4.

selection is  $(2/13, 2/13, 9/13)$ , the equilibrium posterior belief of all other cases is  $(1/3, 1/3, 1/3)$ . Since we asked the experiment participants whose role was a seller to report how likely their matched buyer is of high/middle/low type on an integer scale of 0 to 100, we did not take the prior belief  $(33.3, 33.3, 33.3)$  as a valid answer. Subjects who wanted to submit an equal likelihood in the posterior belief should then submit  $(33, 33, 34)$ ,  $(33, 34, 33)$ ,  $(34, 33, 33)$ , or similar. To count the incidences of the third-order positive selection, we regard any posterior beliefs such that  $\max\{\hat{q}(v_l), \hat{q}(v_m), \hat{q}(v_h)\} - \min\{\hat{q}(v_l), \hat{q}(v_m), \hat{q}(v_h)\} \leq 0.05$  to be close enough to the equilibrium posterior beliefs. Similarly, for the case of  $p_1 = p_H$  in  $M420$ , any posterior beliefs  $\{\hat{q}(v_l), \hat{q}(v_m), \hat{q}(v_h)\}$  such that  $\hat{q}(v_l) \in [0.13, 0.17]$ ,  $\hat{q}(v_m) \in [0.13, 0.17]$ , and  $\hat{q}(v_h) \in [0.66, 0.74]$  are counted as the incidences of the third-order positive selection. However, not many posterior beliefs fall within this generous definition.

**Result 3.** *Only 6.51% of the posterior beliefs (40 out of 614) are rationalized in the third-order positive selection.*

It is worth noting that the third-order positive selection leads the posterior belief to be identical to the prior belief in most cases. Thus, we must keep in mind that some of the observations may not be the result of the third-order reasoning, but the result of zero reasoning. When examining our next result, we will revisit this issue.

Hypothesis 4 regards the individual consistency. From Results 1–3, we know few subjects’ posterior beliefs are consistent with the equilibrium belief. It does not, however, mean that the subjects are irrational. If their second-round price offer is consistent with the optimal price offer derived from their reported belief, then we could say that the sellers’ behavior is properly driven by their incentives to maximize profit.

Overall, 66.94% (411 out of 614) of the second-round price offers were optimal from their subjective beliefs. Among those who passed the first-order positive selection, 70.22% of the second-round price offers were optimal. Among those who pass the second-order positive selection, 76.82% of the second-round price offers were optimal, indicating that higher-order reasoning on positive selection is positively associated with pricing optimality. However, among those who passed the third-order positive selection, only 55.00% of the price offers were optimal. This is because the equilibrium posterior belief  $(1/3, 1/3, 1/3)$  is observationally equivalent to the most naïve subject’s response—keep the prior. Once we regard only those who (1) report the posterior belief closer to the equilibrium posterior and (2) offer the optimal price given their belief in the second round as the “equilibrium sellers,” there are only 22 data points (out of 614), or only 13 subjects out of 147.

**Result 4.** *A majority (66.94%) of the second-round price offers were optimal in the sense that the offer maximizes the expected profit calculated with their subjective beliefs. Higher-order reasoning on positive selection is positively associated with pricing optimality.*

Now we turn to the buyer’s responses. Hypothesis 5 states that even if we observe some

rejections in round 1, such rejections are more likely from the middle- or high-type buyers who receive  $p_1 = p_M$  or  $p_1 = p_H$ . Table 5 shows the proportions of rejected price offers in round 1. Supporting Hypothesis 5, buyers with  $v_M$  or  $v_H$  tend to reject  $p_H$  more frequently than  $p_M$ , and do not reject  $p_L$  at all. What is inconsistent with Hypothesis 5 is that about one third of the low-type buyers rejected  $p_M$  and  $p_H$ . It implies that some low-type buyers expect that the price offer in the second round would sharply drop to  $p_L$ , which is both groundless and worthless. It is groundless because a rational seller who offered either  $p_M$  or  $p_H$  in the first round would never drop the second-round price to  $p_L$ . It is worthless because even in the most optimistic situation where the seller indeed offers  $p_L$  in the second round, the expected payoff  $\delta(v_L - p_L) = 0.8 * 60 = 48$  is strictly smaller than 50, the value of the outside option.<sup>14</sup>

$v \setminus p_1$	$p_L$	$p_M$	$p_H$
$v_L$	9% (1/11)	36% (76/211)	36% (102/280)
$v_M$	0% (0/3)	57% (137/240)	67% (151/227)
$v_H$	0% (0/7)	31% (72/231)	82% (214/260)

Table 5: Proportions of Rejecting the First-Round Offer

Each entry shows the percentage of rejections when the buyer’s type corresponds to the first column, and the seller’s first-round offer corresponds to the first row. The number of rejections over the number of offers is in parentheses.

**Result 5.** *Some buyers with  $v_H$  or  $v_M$  reject the first-round price offer, expecting that the second-round price offer would be more favorable to them. Some low-type buyers also reject price offers where they could have been better off by exercising the outside option.*

Some may argue that the seller’s posterior beliefs may be reflecting the buyers’ (perhaps irrational) behaviors, and that may be why they did not update the posterior belief in an equilibrium way. We still claim that the discrepancies between theoretical predictions and experimental findings are mainly due to the seller’s failure of positive selection reasoning. Note that even if the bargaining process moves to round 2, the seller could not observe the matched buyer’s type, which means that the information presented in Table 5 is unavailable to the seller. Also, even if the sellers correctly anticipated that the proportions of rejecting the first-round price offer are similar to those in Table 5, the ‘empirical’ posterior belief,  $(\hat{q}(v_L|p_1), \hat{q}(v_M|p_1), \hat{q}(v_H|p_1))$  based on such observations would be (29%, 46%, 25%) for  $p_1 = p_M$  and (19%, 36%, 44%) for  $p_1 = p_H$ , which are depicted as blue diamond in Figure 3. Few posterior beliefs are spotted around the empirical posterior belief, implying that it does not seem that the sellers correctly respond to the buyers’ behaviors.

Lastly, we count how many first-round price offers are equal to the equilibrium offer. Overall, 64.29% of the offers (945 out of 1,470) in the first round were the profit-maximizing price. Figure

<sup>14</sup>In the last three periods, such proportions of rejections slightly decrease to 28.36% (19 out of 67) in  $p_M$  and 28.92% (24 out of 83) in  $p_H$ , respectively.

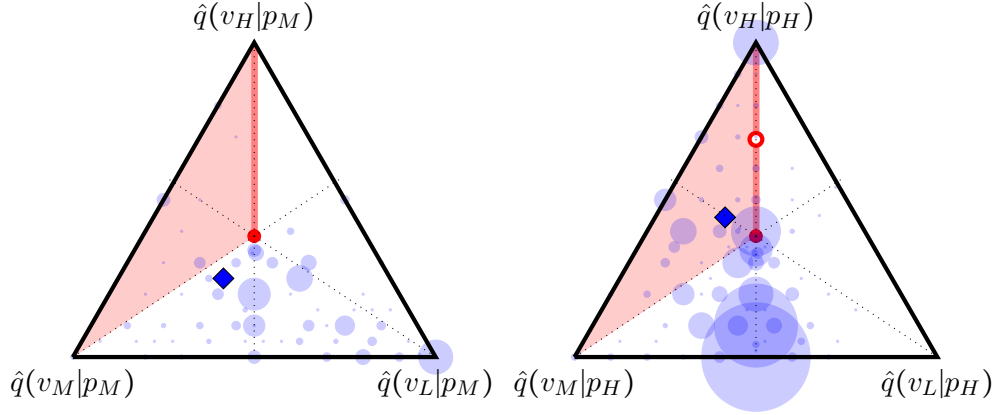


Figure 3: Observed and Empirical posterior beliefs, by  $p_1$

The blue circles on the left (right) depict the reported posterior beliefs after the first-round price offers of  $p_M$  ( $p_H$ ) are rejected. A larger circle means more observations in the center of it. The diamond shape in each simplex indicates the posterior belief consistent with the empirically observed proportions of the first-round price offer rejections.

4 shows the proportion of each price per treatment, and black bars indicate the proportion of the optimal price.

As barely noticeable, few sellers offer  $p_1 = 10$  in all treatments. While in  $M90$  and  $M420$ , the optimal price ( $v_H$  and  $v_M$ , respectively) was most frequently offered, but  $p_1 = 180$  instead of the optimal price was most frequently offered in  $M240$ . Does it mean that the sellers in  $M240$  were less rational? We doubt this. It is more plausible that the experiment participants are inclined to offer the middle price if that middle one is not too far from what they expect to be optimal.

## 5 Conclusion

In this paper, we experimentally examine how the idea of positive selection unfolds in a two-round bargaining game with one-sided, incomplete information. Inconsistent with theory, a substantial proportion of first-round price offers are rejected when an outside option is available. The reported beliefs from the sellers after a price offer is rejected confirm that the higher-order positive selection does rarely take place in the lab: Even in our generous definition, only about 7% of the posterior beliefs are considered as the result of the third-order positive selection. Worse yet, about half of those with the “correct” posterior belief were the most naïve ones who merely stuck to the prior and could not find the profit-maximizing price. Overall, this failure of the inductive process of positive selection leads the sellers to earn profits much less than what the theory predicts.

The main contribution of this paper is to decipher at which level of positive selection reasoning fails. This is beyond merely examining the validity of the theoretical predictions of the game, and we believe it is more important for counterfactual analyses and policy suggestions. We found a

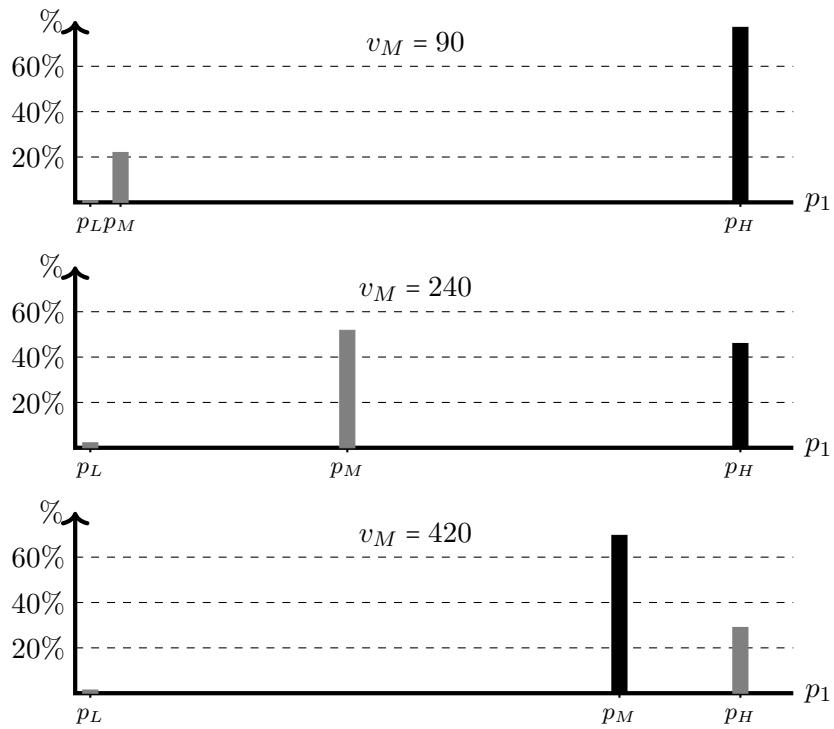


Figure 4: Optimal pricing or targeting the middle

Each bar depicts the percentage of the first-round offers. The black bar indicates the optimal price offer given  $v_M$ .



clear rejection of the theoretical predictions, and our experimental design allows us to enunciate the drivers of the discrepancies between experimental data and theoretical predictions. We hope our method of experimentally stripping down the several layers of rational reasoning to be used in various contexts whose equilibrium predictions are derived from multiple thought processes.

Unraveling serves as a fundamental concept in numerous theoretical findings across different contexts, such as bilateral bargaining and information disclosure. Our study suggests that the observed failures of unraveling in both laboratory and real-world settings ([Jin et al., 2021](#); [Brown et al., 2012](#)) may primarily stem from a lack of strategic sophistication among participants. This raises the question of whether any intervention aimed at facilitating the initiation of the unraveling process could potentially trigger the entire process. To the best of our knowledge, no prior research has investigated the responsiveness of unraveling to such interventions at the initial stage. We consider this question to be an intriguing avenue for future research.

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## A Proof of Proposition 1

We focus on the case  $\epsilon$  is positive but small; the case  $\epsilon = 0$  is already proved in the main text. Pick any sequence of equilibria  $((\sigma^n, \tau^n, \hat{q}^n))_{n=1}^\infty \in \prod_{n=1}^\infty \mathcal{E}(\epsilon^n)$  such that  $\epsilon^n > 0$  for all  $n$  but vanishes to zero as  $n \rightarrow \infty$ . It suffices to show that

$$\lim_{n \rightarrow \infty} \sigma_v^n(D|p_1) = 1 \quad \forall p_1 \in \mathcal{P}, v \in V.$$

Pick  $p_1 \in \mathcal{P}$  arbitrarily, and suppose for contradiction that there is a buyer type  $v \in V$  such that  $\limsup_{n \rightarrow \infty} \sigma_v(D|p_1) > 0$ . Let  $v_0$  be the lowest one among all such buyer types; that is,  $\lim_{n \rightarrow \infty} \sigma_v(D|p_1) = 0$  for any  $v < v_0$ . We may assume without loss that  $\sigma_v^n(D|p_1)$  is convergent for all  $v$  (we may take a subsequence, if necessary). Now, suppose that the seller offers  $p_1$  in the first round and then this offer is rejected by the buyer. Also, suppose that the negotiation proceeds to the next round, and then, the seller offers  $p_2 = p'_2 < \bar{p}(v_0)$  in the second round.  $p'_2$  is accepted by the buyer if and only if  $u(v) > p'_2$ , and thus, the seller's payoff from charging  $p'_2$  is

$$\Pi^n(p'_2) := p'_2 \sum_{v:u(v)>p'_2} \hat{q}^n(v).$$

Similarly, the seller's payoff from charging  $p_2 = \bar{p}(v_0)$  in the second round is

$$\Pi^n(\bar{p}(v_0)) := \bar{p}(v_0) \sum_{v:u(v)>\bar{p}(v_0)} \hat{q}^n(v).$$

The difference between these two payoffs is

$$\begin{aligned} \Pi^n(\bar{p}(v_0)) - \Pi^n(p'_2) &= (\bar{p}(v_0) - p'_2) \sum_{u(v)>\bar{p}(v_0)} \hat{q}^n(v) - p'_2 \sum_{p'_2 < u(v) < \bar{p}(v_0)} \hat{q}^n(v) \\ &> (\bar{p}(v_0) - p'_2) \hat{q}^n(v_0) - p'_2 \sum_{p'_2 < u(v) < \bar{p}(v_0)} \hat{q}^n(v) \\ &= \hat{q}^n(v_0) \left[ \bar{p}(v_0) - p'_2 - p'_2 \sum_{p'_2 < u(v) < \bar{p}(v_0)} \frac{\hat{q}^n(v)}{\hat{q}^n(v_0)} \right] \end{aligned}$$

where the inequality holds because  $\bar{p}(v_0) > p'_2$  and  $v_0$  is the smallest buyer type such that  $u(v) > \bar{p}(v_0)$ . By Bayes' rule,

$$\lim_{n \rightarrow \infty} \frac{\hat{q}^n(v)}{\hat{q}^n(v_0)} = \lim_{n \rightarrow \infty} \frac{q(v)[\epsilon^n + (1 - \epsilon^n)\sigma_v^n(D)]}{q(v_0)[\epsilon^n + (1 - \epsilon^n)\sigma_{v_0}^n(D|p_1)]} = \frac{q(v) \cdot 0}{q(v_0) \lim_{n \rightarrow \infty} \sigma_{v_0}^n(D|p_1)} = 0$$

for any  $v$  such that  $u(v) < \bar{p}(v_0)$ . On the other hand,

$$\begin{aligned} \lim_{n \rightarrow \infty} \hat{q}^n(v_0) &= \lim_{n \rightarrow \infty} \frac{q(v_0)[\epsilon^n + (1 - \epsilon^n)\sigma_{v_0}^n(D|p_1)]}{\epsilon^n + (1 - \epsilon^n)\sum_v q(v)\sigma_v^n(D|p_1)} \\ &> \lim_{n \rightarrow \infty} q(v_0) \left( \epsilon^n + (1 - \epsilon^n)\sigma_{v_0}^n(D|p_1) \right) = q(v_0) \lim_{n \rightarrow \infty} \sigma_{v_0}^n(D|p_1) > 0 \end{aligned}$$

where the inequality holds because the denominator  $\epsilon^n + (1 - \epsilon^n)\sum_v q(v)\sigma_v^n(D|p_1)$  is always weakly less than 1. Thus,

$$\lim_{n \rightarrow \infty} \left( \Pi^n(\bar{p}(v_0)) - \Pi^n(p'_2) \right) > \lim_{n \rightarrow \infty} \hat{q}^n(v_0)(\bar{p}(v_0) - p'_2) > 0 \quad \forall p'_2 < \bar{p}(v_0).$$

This shows that, for any  $n$  such that  $\Pi^n(\bar{p}(v_0)) - \Pi^n(p'_2) > 0$  (which is the case for all sufficiently large  $n$ ), the seller will never charge  $p'_2 < \bar{p}(v_0)$  in the second round. Thus, the buyer type  $v_0$ 's expected payoff from rejecting  $p_1$  and delaying the negotiation to the second round is at most

$$\delta \max\{w, v - \bar{p}(v_0)\} < \delta \max\{w, w + \Delta\} = \delta(w + \Delta) < \delta w + (1 - \delta)(1 - \epsilon^n)w \leq w$$

where the first and second inequalities respectively follow the definition of  $\Delta$  and (A2). The buyer type  $v_0$  finds it strictly more profitable to exercise the outside option in the first round than delaying the negotiation. This contradicts the hypothesis that  $\limsup_{n \rightarrow \infty} \sigma_{v_0}^n(D|p_1) > 0$ .

## B Sample Experimental Instructions

### INSTRUCTIONS (For $v_M = 240$ )

Welcome to the experiment. Please read these instructions carefully. There will be a quiz around the end of the instructions, to make sure you understand this experiment. The payment you will receive from this experiment depends on your decisions.

#### Your Role and Match

At the beginning of the experiment, one-half of the participants will be randomly assigned to the role of a **seller** and the other half the role of a **buyer**. Your role will remain fixed throughout the experiment.

The experiment consists of 10 **matches**. At the beginning of each match, one seller participant and one buyer participant are randomly paired. The pair is fixed **within the match**. After each match, participants will be reshuffled to form new pairs. You will not learn the identity of the participant you are paired with, nor will that participant learn your identity—even after the end of the experiment.

In this experiment, there is a seller who has an asset with no value, and a buyer who values the asset positively. The buyer's value of the asset is represented by  $B$ . At the start of each match, the computer randomly selects  $B$  from the set  $\{70, 240, 500\}$ , where each value is equally likely to be chosen.  $B$  is fixed for each match but is independently selected for each new match. Importantly, **the buyer knows the value  $B$ , but the seller does not.**

### Your Decisions in Each Match

Each match consists of up to **two rounds** of bargaining. In Round 1, a seller offers a price to sell the asset, and the buyer responds. If the offer is rejected, the match may move on to Round 2 of bargaining. The details follow.

**Your Task as a Seller in Round 1:** Suppose your role is a seller. At the beginning of Round 1, you will see the following figure. Three blue bars between 50 and 500 represent the possible values of  $B$ .

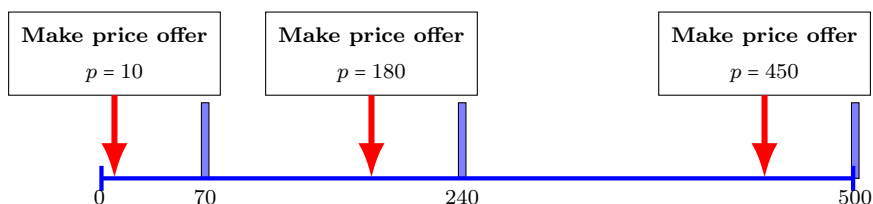


Figure 5: Seller's Screen: Round 1

Choose your price offer by clicking one of the three buttons: “Make price offer  $p = 10$ ”, “Make price offer  $p = 180$ ”, and “Make price offer  $p = 440$ ”. After that, click the submit button, and wait for the buyer’s decision. You expect one of three possible outcomes.

- If the buyer **accepts** the offer, the match is over and you earn  $p$  tokens.
- If the buyer **takes an outside option**, the match is over and you earn 0 tokens.
- If the buyer **rejects** the offer, then the match moves to Round 2 with an 80% chance. Note that if the match is terminated with a 20% chance, both you and the buyer earn 0 tokens.

When the buyer accepts the offer or takes an outside option, the buyer’s decision will be correctly carried with a 99.9% chance. With a 0.1% chance (1 in 1,000), however, a server computer overrides the buyer’s decision and rejects the offer. This minuscule probability is merely introduced to preserve a *theoretical* possibility of reaching Round 2. Since this probability is negligible, overriding is unlikely to happen in the course of your participation.

**Your Task as a Buyer in Round 1:** Suppose your role is a buyer. At the beginning of Round 1, you will see the following figure. The horizontal position of the blue bar represents  $B$ , your value of the asset. ( $B$  is

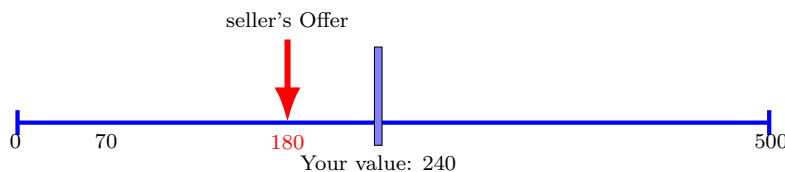


Figure 6: Buyer's Screen in Each Round

240 in this example, but your value can be 70 or 500 as well.) Once the seller in your pair offers a price,  $p$ , a red vertical arrow will appear on the figure. The position of the red arrow represents  $p$ . After that, decide whether to

- **accept** the offer and earn  $(B - p)$  (in which case the match is over),
- **reject** it and move on to the next round with an 80% chance, or
- **take an outside option** to earn 50 tokens (in which case the match is over).

Beware that you cannot accept the offered price  $p$  if it is strictly greater than your value  $B$ , otherwise your payoff becomes negative. When you accept the offer or take an outside option, your decision will be correctly carried with a 99.9% chance. With a 0.1% chance (1 in 1,000), however, a server computer overrides your decision and rejects the offer. This minuscule probability is merely introduced to preserve a *theoretical* possibility of reaching Round 2. Since this probability is negligible, overriding is unlikely to happen in the course of your participation.

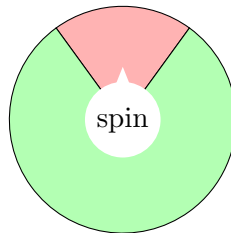


Figure 7: Spinning Wheel

**Transition to Round 2:** As described, when bargaining does not end in Round 1, the match continues to Round 2 with an 80% chance. Your screen presents a spinning wheel that consists of red area (20%) and green area (80%) as illustrated below. Once you click the Spin button, the wheel starts spinning. If the spinning wheel stops at the green area, the match continues. Otherwise, the match terminates. Note that the seller and buyer in the same match always see the same outcome from the wheel.

**Your Task as a Seller in Round 2:** Before submitting a new price offer, report how you believe the buyer's value, by filling out the following sentence.

I believe that the value of the buyer paired in this match is

70 with	a (----)% of chance,
240 with	a (----)% of chance,
500 with	a (----)% of chance.

The three numbers must sum up to 100. **The reported probabilities will appear in your decision screen but will not be shared with the buyer.**

Its sole objective is to help you think about an appropriate price offer. It is important to note that there are no advantages to indicating probabilities that differ from your true belief, so please report your belief as accurately as possible.

Note that the buyer's value of the asset ( $B$ ) and the value of the outside option (50 tokens) will remain the same across rounds within a match.

**Your Task as a Buyer in Round 2:** Your screen will present the same figure as the one you had in Round 1 that indicates your value. Once the seller in your pair makes a price offer in Round 2, a red vertical arrow will appear on the figure. The position of the red arrow represents  $p$ . After that, decide whether to

- **accept** the offer and earn  $(B - p)$ ,
- **reject** it and earn 0 tokens, or
- **take an outside option** to earn 50 tokens.

Beware that **the match will be over at the end of Round 2.**

### Information Feedback

- At the end of each **round**, you will know the seller's price offer and the buyer's decision. If the buyer rejects the offer, you will know whether the match is continued to the next round or terminated.
- At the end of each **match**, you will know how many tokens you receive from the match.

### Your Monetary Payments

At the end of the experiment, a computer will randomly select one match out of 10 for your payment. Every match has an equal chance to be selected for your payment, so it is in your best interest to take each match equally seriously. Participants will receive the amounts of tokens according to the outcome from the selected match with the exchange rate of 1 token = 1 HKD. Also, every participant will receive a **show-up fee of HKD 40.**

### Completion of the Experiment

After the 10th match, the experiment will be over. You will be instructed to fill in the receipt for your payment. The amount you earn will be paid **electronically via the HKUST Autopay System to the bank account you provide to the Student Information System (SIS).** The Finance Office of HKUST will arrange the auto-payment. An email notification will be sent to your HKUST email address on the pay date under the name of the sender "FOPSAP" (Finance Office Payment System Auto Payment).

### Comprehension Check

To ensure your comprehension of the instructions, you will answer four multiple-choice questions. You can proceed only with all correct answers. Afterwards, you will participate in a practice match.

- Q1 Suppose you are a seller. Which of the followings is NOT TRUE? (a) I do not know how much the buyer values the asset. (b) If the buyer takes an outside option, I earn 50 tokens. (c) If I offer 440 tokens, and the buyer accepts it, then I earn  $45p$  tokens. (d) If bargaining does not end in Round 1, then I can make a new offer with a 80% chance.
- Q2 Suppose you are a buyer, and the value of the asset is 500. Which of the followings is TRUE? (a) If I accept a price offer of 180 tokens, I earn 180 tokens. (b) If I take an outside option, I earn 550 tokens. (c) If I accept a price offer of 180 tokens, I earn 320 tokens. (d) In Round 2 of this match, the value of the asset will be different from 500.



- Q3 Suppose the price offer in Round 1 is rejected. Which of the followings CAN HAPPEN? (a) The match is terminated, and both the seller and the buyer earn 0 tokens. (b) The match is continued forever, even after continuous rejections. (c) The match is terminated, and each participant in the pair earns a half of value  $B$ . (d) The match initiates an open chat to negotiate.
- Q4 Suppose the first match is done. Which of the followings is TRUE? (a) It is almost sure that I will be paired with the same participant in the first match. (b) I may play another role different from what I did in the first match. (c) The buyer's value of the asset in the second match will be the same as the one in the first match. (d) My previous actions do not affect the value of the asset in the new match.

### **1 Practice Match and 10 Actual Matches**

Thank you for paying attention to the instructions. Before you will play the 10 actual matches, you will have one practice match (Match #0) which is not relevant to your payment. Its objective is to get you familiar with the computer interface and the flow of the decisions in each round of a match. Once the practice match is over, it moves to the actual matches.