Population Uncertainty in Voluntary Contributions of Public Goods☆

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Abstract

I examine how uncertainty in the size of the relevant population affects the voluntary contribution of public goods. I analyze a case where the marginal production of public goods is decreasing and convex, and agents' social preferences are irrelevant to the population size. The voluntary contribution level in Nash equilibrium is higher when the number of players is random than when the number of players is fixed at the mean of the population distribution. The findings from a controlled experiment are consistent with this theoretical prediction. I also analyze a case where the production function is linear and the agents' social preferences are modeled in the form of a warm-glow utility function which could be increasing concave in the population size. The experimental findings reject the hypothesis that warm glow is congestible: When the public-goods production function is linear, uncertainty in the population size does not lead to changes in the contribution level.

JEL Classification: C72, D64, H41

Keywords: Population uncertainty, Voluntary contributions mechanism, Public goods, Laboratory experiment

1. Introduction

I examine how individuals contribute to the production of public goods when they do not know the exact number of participants in the contributor pool. Previous research has used lab experiments to determine what factors encourage individuals to contribute to the provision of

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public goods.[1] The voluntary-contributions mechanism (VCM), including variations thereof, has been repeatedly revisited by verifying observations,[2] extending ideas,[3] or extrapolating results[4] from the existing literature. One common result found in the vast majority of previous studies is that some subjects seem to feel a "warm glow," which can be modeled by additional utility derived from the very act of giving (Cornes and Sandler, 1984; Andreoni, 1989, 1990). In terms of experimental design, another noticeable similarity of previous studies is that all experiment participants knew exactly how many other individuals were making their decisions simultaneously, and accordingly knew exactly how influential their contributions were. However, in many real-world situations, a potential contributor does not know how many other contributors there are: A voter does not know how many people with voting rights will consider turning out for an election, a charitable giver does not know how many others will consider making donations of aid to needy children, a voluntary participant in a Neighborhood Watch group does not know how many neighbors would consider filling in—on short notice—for members who are out of town during the summer vacation period, and potluck party organizers do not know how many others will contribute since they do not know how many will consider coming.[5] Does this uncertainty change the behavior of individuals? If so, how does the uncertainty change it?

[1] Experimental studies of private provision of public goods include, but are not limited to, Marwell and Ames (1979), Isaac et al. (1994), Smith et al. (1995), Palfrey and Prisbrey (1997), Isaac and Walker (1988), Bagnoli and McKee (1991), Fehr and Gächter (2000), and Croson (2007).

[2] Andreoni (1995a) found that, on average, about half of all voluntary contributions in the laboratory comes from subjects who understand that non-cooperation maximizes their payoffs but choose to cooperate out of some form of kindness. Brandts and Schram (2001) found that subjects' behavior cannot be explained exclusively as the result of errors in making choices. Fischbacher et al. (2001) found in a one-shot public-goods game that half of the subjects were conditional cooperators. Harbaugh and Krause (2000) conducted a public-goods game with children and found that older children's behavior was similar to that of adults.

[3] Andreoni (1995b) found asymmetries in subjects' behavior between provision of public goods and provision of public "bads." Messer et al. (2007) studied how contextual factors can produce sustained efficiencies in a voluntary-contribution game. Morgan (2000) considered a way to increase contributions by introducing a certain feature of lotteries, and Morgan and Sefton (2000) tested the idea experimentally. Zhang and Zhu (2011) investigated the effect of group size in a natural field experiment via the Chinese Wikipedia site.

[4] Andreoni (1993) and Andreoni and Payne (2003) tested the proposition that government contributions via lump-sum taxation will completely crowd out voluntary contributions to the production of public goods and found that such crowding-out was incomplete. Fehr and Gächter (2000) and Masclet et al. (2003) added some forms of punishment for people who did not contribute voluntarily. Duffy et al. (2007) and Fischbacher and Gächter (2010) investigated how the dynamics of public-goods games affected subjects' behavior. Keser and Van Winden (2000) and Andreoni and Croson (2008) studied whether subjects' behavior changes when they play with partners instead of with strangers (or vice versa).

[5] I distinguish the uncertainty in the number of players from the uncertainty in how a known number of other players would act. I will discuss further the distinction between population uncertainty and changes in population size.

The main contribution of this paper is to provide theoretical and experimental evidence that population uncertainty is one of the driving factors behind voluntary contributions. If the production function of the public goods is increasing concave, individuals contribute more when the population size is uncertain than when the population size is fixed at the mean of the population distribution. The warm glow utility, at least for the group sizes being tested, doesn't get affected by population uncertainty.

To be more specific, I modeled a voluntary contribution game with population uncertainty. A player in this game knows the population distribution of the players but not the exact number of players. This randomness is referred to as population uncertainty. Though population uncertainty has been adopted in many other fields in applied microeconomics,[6] to the best of my knowledge it has not been emphasized in the literature on voluntary contributions of public goods.

Two issues on which consensus has not been reached are (1) whether voluntary contributions increase or decrease with group size, and (2) whether people strategically contribute conditional on the structure of public-goods production. To answer them efficiently, I considered two models, each of which closes off one channel through which population uncertainty could affect the players' contribution decisions. With a model that features an increasing concave public-goods production function, with a convex marginal production[7] and a constant marginal warm-glow utility function in terms of population size, I show that when the number of players is random, the voluntary contribution level in a symmetric Nash equilibrium is *higher* than when the number of players is fixed at the mean number of players. Another model focuses more on the warm-glow utility (Cornes and Sandler, 1984; Andreoni, 1989, 1990). If the warm-glow utility can be described by a function which is increasing concave in the group size and the marginal utility of public goods is constant in the group size, then, unlike with the first model, population uncertainty serves to *decrease* the individual contribution level in equilibrium. However, this prediction depends strongly on the assumption made about the shape of the warm-glow utility function in the group size.

While these theoretical predictions of the effect of population uncertainty for each model

[6] Since Myerson (1998b) introduced the notion of population uncertainty in games, studies in political economy have actively used the concept of population uncertainty to understand voter turnout (Myerson, 1998a; Piketty, 2000; Dhillon and Peralta, 2002; Bendor et al., 2003; Spenkuch, 2013). In the context of contests, population uncertainty has also played an important role (Myerson and Wärneryd, 2006; Münster, 2006; Lim and Matros, 2009).

[7] It is known that public goods provision with diminishing marginal returns yields an interior solution. See, for example, Keser (1996), Sefton and Steinberg (1996), and Laury et al. (1999).

are evident, it is still unclear how people actually respond to population uncertainty. Though the two channels I considered are undoubtedly important, there could be other factors that affect the individual contribution level under population uncertainty. If some players regard the uncertainty in the population size as a cognitive barrier that hampers them from calculating a strategically optimal contribution, they might want to increase their allocation to the consumption of private goods so that their utility will come from a more certain source. On the other hand, if risk-averse subjects worry more about the worst-case scenario, where the population size turns out to be small, or their risk aversion drives them to put more weight on the possibility that the contributor pool is small, they may want to contribute more in order not to forgo the higher marginal utility of public goods. Another possibility is that the salience of population uncertainty would change a subject's decision process, by implicitly encouraging him to recognize the strategic aspects of the game. My laboratory experiments are designed to correspond to my models, but they also serve to examine whether certain other factors affect the individual contribution level.

I conducted a series of experiments designed to test hypotheses about how population uncertainty affects voluntary contributions of public goods. I employed a 2-by-2 between-subject treatment design. The four treatments differed in two dimensions: the functional form of public-goods production (linear or nonlinear) and where the group size was certain or uncertain. That is, the treatment effect of the group-size uncertainty was examined both with a linear production function and with one that is increasing and concave. Subjects in the uncertainty treatment chose their contribution level without knowing the actual group size, which was either 3 or 9 with equal probability. Their earnings in each round were determined after the group size was revealed. The linear-treatment sessions investigated whether and how people respond to population uncertainty in the linear VCM, where production of public goods is linear in total contributions. The basic structure of the linear–certainty experiment resembled the standard VCM experiment, such as that of Palfrey and Prisbrey (1997). To minimize the effects of dynamic strategies, I adopted random rematching (Andreoni and Croson, 2008).[8]

The experimental results can be summarized as follows: (1) With the nonlinear production function, the average contribution to the group account is larger when the group size

[8]When subjects are matched with the same group members for multiple periods, they have an incentive to behave strategically: If they can send credible signals in some rounds that they are cooperative, then they can leverage their earnings by betraying the other members of their group and behaving non-cooperatively in the final round.

is uncertain than when it is certain. (2) No significant differences in the contribution levels are observed with the linear production function. This observation does not support the hypothesis that individuals' social preference regarding the very act of making a contribution, which can be captured by an additive utility term, is increasing concave in the group size. (3) Gender, ethnicity, and risk preferences were not significant factors in determining the contribution level.

The rest of this paper is organized in the following way. In the following subsection I discuss the closely related literature. Section 2 describes a simple model, and Section 3 presents theoretical findings that will shed light on the experimentally testable hypotheses. Section 4 describes the design and procedure of the experiments, and Section 5 highlights the experimental results. Section 6 discusses some potential extensions of this study, and Section 7 concludes.

1.1. Related Literature

This study extends previous studies that addressed the relationship between population size and pro-social behavior. Many studies following Isaac et al. (1984) and Isaac and Walker (1988) have considered relationships between free-riding behavior and group size. Isaac et al. (1994) found that groups of size 40 or 100 provided a public good more efficiently than groups of size 4 or 10, while standard theory predicts the opposite. Goeree et al. (2002) also found that contributions are generally increasing in the group size. Carpenter (2007) found that people will punish free riders—even at considerable cost—and that such punishment does not fall appreciably in large groups. Nosenzo et al. (2013) found that patterns of increased contributions with group size are observed only when the marginal per capita return on private consumption is low. However, the difference between population uncertainty and changes in population size should be clarified, because each economic agent's outcome could be uncertain even when the population size is fixed. In the context of this paper, the unknown strategic behavior of other agents, which can be refined by many strategic equilibrium concepts, is not regarded as population uncertainty. By population uncertainty, I mean only the randomness in the population size. Population uncertainty is described in a manner similar to that of Poisson games (Myerson, 1998b; Myerson and Wärneryd, 2006), but my models do not require having unbounded support.

This paper also addresses changes in the warm glow utility regarding the size of potential recipients of the contribution. In this regard, Andreoni (2007) and Exley (2016) are worth being discussed. Andreoni (2007) points out that voluntary contribution (in a pure altruistic sense) constitutes a congestive public good because an increase in the group size decreases the price of providing a unit of social value and at the same time reduces individuals' incentives to contribute. He found that for the average subject, a gift that results in one person receiving g is equivalent to n people receiving $g/n^{0.68}$ each, that is, for most subjects altruism ($gn^{0.32}$) is increasing and convex in the population size. I take this finding as one axis of model setups. Exley (2016) found that charity donors give less when the impact of their contributions is uncertain, which could be consistent with the theoretical prediction for a model with an increasing concave utility of giving. Though the uncertainty of the impact of the contributions studied in Exley (2016) is different to population uncertainty, there could be some relationships between them: An uncertain population size implies an uncertain number of recipients of the contribution. If the economic agent only cares for the social value that she/he creates by making a contribution, population uncertainty may drive smaller contributions.

2. The Model

As a benchmark, consider a case where the number of players is fixed. Let $N = \{1, 2, \ldots, n\}$, $n \geq 2$, denote a set of homogeneous potential contributors. Each contributor has endowment $y > 0$. There are one public good and one private good. Contributor $i \in N$ voluntarily contributes $g_i \geq 0$ to the provision of the public good and consumes the remainder, $x_i = y - g_i$. The supply function for the amount of the public good which is produced as a result of the voluntary contributions, $f : \mathbb{R}_+ \to \mathbb{R}_+$, is an increasing, weakly concave function of the sum of the individual contributions, $G = \sum_{i=1}^{n} g_i$. The utility function for contributor i is $U(x_i, g_i, G; n) = x_i + \psi(g_i, n) + f(G)$, where $\psi(\cdot, \cdot)$ captures the warm-glow utility (Cornes and Sandler, 1984; Andreoni, 1989, 1990), which may depend on the population size. $\psi(\cdot, \cdot)$ is weakly increasing on $\mathbb{R}_+^2$ and differentiable on $\mathbb{R}_{++}^2$. This additively separable quasi-linear form is assumed for simplicity, and relaxation of this assumption does not change the direction of the main theoretical findings.

In a game with population uncertainty, the number of individual players is random. Formally, let $N_{1+} = \{2, 3, \ldots\}$, and let $\pi : N_{1+} \to [0, 1]$ be the commonly known probability density function for the number of players (hence $\sum_{n=2}^{\infty} \pi(n) = 1$). Let $\mu := \sum_{n=2}^{\infty} \pi(n)n$, that is, the expected number of players. When π is a degenerate function with $\pi(n) = 1$, this setup reduces to the benchmark. I assume risk neutrality with respect to private consumption, because in the context of a voluntary contribution game, x simply represents the risk-neutral monetary value.

Consider this voluntary contribution game from the perspective of a single player.[9] He is told that the actual population size will be from a distribution whose density function is $\pi(n)$.[10] This player's objective is to choose the contribution level g that maximizes his expected utility given the contribution of any other player (which is presumed to be symmetric in the players), denoted by $\tilde{g}$.

Definition 1. *A contribution level g^* is a symmetric equilibrium if the following hold:*
(i) $g^* = \arg\max_{g\in[0,y]} \sum_{n=2}^{\infty} \pi(n)\{y - g + \psi(g,n) + f(g + (n-1)\tilde{g})\}$
(ii) $g^* = \tilde{g}$.

To exclude the trivial equilibrium where everyone contributes nothing, assume that some positive amount of the public good is desirable.

Assumption 1. $\lim_{g\to 0}[\psi_1(g,n) + f'(g)] > 1$,

where $\psi_1(g,n) = \partial\psi(g,n)/\partial g$. Assumption 1 states that even when no other players contribute, it is desirable to contribute a nonzero amount of one's endowment to the production of public goods. Note that the right-hand side of the inequality is the marginal cost of increasing the contribution in the case of a quasi-linear utility function, so this assumption could be generalized to $\lim_{g\to 0} U_2 + f' > \lim_{g\to 0} U_1$, where U_i is the partial derivative of U with respect to the ith argument.

Assume further that y is large enough that no two players should contribute their entire endowments.

Assumption 2. $\lim_{g\to y}[\psi_1(g,n) + f'(y+g)] < 1$

Assumption 2 states that contributing all of one's endowment is not desirable when another player contributes his/her entire endowment to the production of the public good.

[9]Since the population size in this model is always at least 2, we can assume that there always exists a player and his/her match participating in this game.

[10]Some studies, especially in auction theories, deal with population uncertainty by introducing a pre-stage where some players are selected from the entire population pool and then they play the game. In such a setup a single player should update his belief conditional on the fact that he is being selected. In this paper, $\pi(n)$ can be understood as a posterior belief that has already been updated. Also, in this context the players' participation within the realized population is not their choice variable, unlike in other studies of voluntary participation, including Dixit and Olson (2000) and Saijo and Yamato (2010).

3. Analysis

In the following two subsections, I close off one channel through which population uncertainty could affect the individual contribution level in equilibrium. In the course of doing so, the role played by population uncertainty will be made clearer.

3.1. When $\psi(g,n) = \psi(g)$

This section studies one model where the marginal utility of warm glow is unaffected by population size, that is, $\psi(g,n) = \psi(g)$. This restriction will be relaxed in the following section. The player's maximization problem simplifies to

$$\max_{g_i \in [0,y]} \sum_{n=2}^{\infty} \pi(n)(y - g_i + \psi(g_i) + f(g + (n-1)\tilde{g})$$

By Assumptions 1 and 2, corner solutions can be excluded, so the first-order condition is

$$\sum_{n=2}^{\infty} \pi(n) \left[\psi'(g^*) + f'(g^* + (n-1)\tilde{g})\right] = 1. \tag{1}$$

In the symmetric equilibrium, where $g^* = \tilde{g}$, the first-order condition becomes

$$\sum_{n=2}^{\infty} \pi(n) f'(ng^*) + \psi'(g^*) = 1. \tag{2}$$

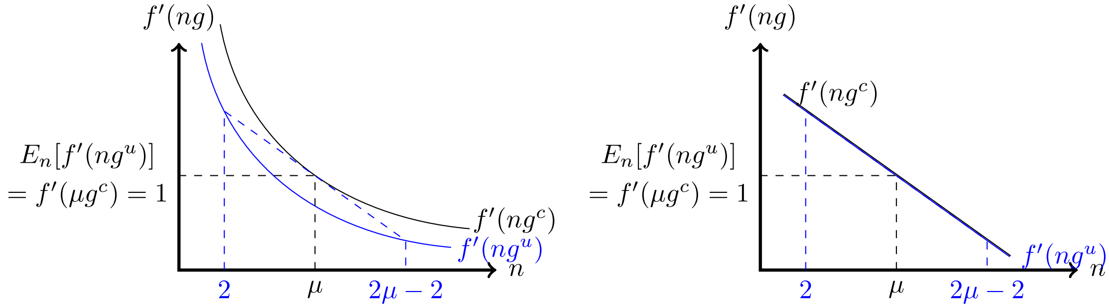
Without population uncertainty, the first-order condition is

$$f'(\mu g^*) + \psi'(g^*) = 1, \tag{3}$$

which is the same as the standard voluntary contribution model's optimality condition that was shown, for example, by Bergstrom et al. (1986) for the case where the warm-glow utility is nonexistent or $\psi'(\cdot) = 0$. The existence and uniqueness of the symmetric voluntary contribution equilibrium are well established in Cornes (2009). Since $\sum_{n=2}^{\infty} \pi(n) n g^* = \mu g^*$, we can read equation (3) as $f'(\sum_{n=2}^{\infty} \pi(n) n g^*) + \psi'(g^*) = 1$. Then by Jensen's inequality it immediately follows that the equilibrium contribution level with population uncertainty is greater than the level with certainty if the marginal production function is convex. Figure 3.1 illustrates Proposition 1.

Proposition 1. *Suppose $f'(\cdot)$ is (weakly) convex and $\psi(g,n) = \psi(g)$. Let g^u and g^c denote the equilibrium contribution levels with and without population uncertainty, respectively.*

Figure 1: An Illustration of Proposition 1



These figures illustrate a special case of population uncertainty, where the population is either 2 or $2\mu - 2$ with equal probability and $\psi'(\cdot) = 0$. (Left) When $f'(ng^c) > f'(ng^u)$, the individual contribution level under population uncertainty, g^u, is larger than the contribution level when the population is certain, g^c. This requires the marginal production function to be convex. (Right) If the marginal production is linear, population uncertainty plays no role.

Then $g^u \geq g^c$, with equality if and only if $f'(\cdot)$ is linear.

Proof: See Appendix B.

The shape of the public goods production function is the main driving force behind the result of Proposition 1. Though in many previous studies a constant marginal production has been considered, I claim that convexity of marginal production is more generally accepted than constant marginal production.[11] Conventional production functions, such as $\ln g$ and $g^\alpha, \alpha \in (0, 1)$, satisfy the condition of convexity of marginal production. Note that $g^u \geq g^c$ holds with equality when the marginal production is linear, that is, either when the production function is quadratic or when it is linear.

The intuition behind Proposition 1 is closely related to an opposite finding by Myerson and Wärneryd (2006) for contests: The aggregate level of effort (investments in a war of attrition, or bids in an all-pay auction) is smaller when the population size is uncertain than when it is certain. Both in contests for private prizes and in voluntary contributions of public goods, each individual's marginal cost of investment is known to her/him with certainty. The population uncertainty plays a role in the marginal benefit of investment. In contests where payoffs are given only to the winners, players are reluctant to exert additional

[11]Admittedly, not every increasing concave function has a positive third-order derivative. For example, $f(g) = \alpha g - g^2, \alpha > 0$, is concave on $\mathbb{R}$, and $f'(g) = \alpha - 2g$, which is not convex. In general, however, such functions are increasing only up to some point, and then decreasing thereafter. In the example, the production function is decreasing when g is greater than $\alpha/2$.

effort, because their marginal benefit will be small if the population turns out to be large. In voluntary-contribution games where payoffs are distributed to all the participants, players are encouraged by the possibility of a small population, which will render their contributions more influential, even to themselves. To put it differently, in a voluntary-contribution game, subjects will be more sensitive to the possibility of the group size being small. I utilized this idea in the laboratory experiments.

If the warm-glow utility from a contribution is additively separable from the utility of the public goods provided and does not depend on the population size, the conclusion of Proposition 1 holds regardless of the existence of the warm-glow utility. Thus individuals' pro-social behavior may be decomposed into their warm glow[12] and their response to population uncertainty. The first step in investigating the validity of such a result is to check whether the warm-glow utility does depend on population uncertainty. We can utilize the standard linear VCM here, because with this linear production function, population uncertainty plays no role in and of itself (Proposition 1). Responses to population uncertainty in the case of a linear production function tell us whether (and how) the warm-glow utility depends on population size. Following the assumptions made in Andreoni (2007), I assume that $\psi(g, n)$ can be factorized by a nondecreasing function of g and a function of n which may be increasing concave in n.[13] In this case, the signs of $\frac{\partial\psi(g,n)}{\partial n}$ and $\frac{\partial\psi^2(g,n)}{\partial g \partial n}$ are always identical, so are the signs of $\frac{\partial\psi^2(g,n)}{\partial n^2}$ and $\frac{\partial\psi^3(g,n)}{\partial g \partial n^2}$. In words, with these assumptions, the warm glow is concave in n if and only if the marginal warm glow (with respect to g) is concave in n. For notational simplicity, let $\phi(g, n) = \frac{\partial\psi(g,n)}{\partial g}$.

3.2. When $\partial\phi(g, n)/\partial n > 0$ and $\partial^2\phi(g, n)/\partial n^2 < 0$

Taking findings from Andreoni (2007) as a starting point, I assume here that $\phi(g, n)$ is increasing and concave in n and the production function is linear, that is, $f'(\cdot) = k$, where k is a constant. Then the conclusions of Proposition 1 do not hold, and an individual's voluntary contribution level decreases with population uncertainty.

Proposition 2. *Suppose $f'(\cdot) = k$, $\partial\phi(g, n)/\partial n > 0$, and $\partial^2\phi(g, n)/\partial n^2 <$ (resp. $>, =$) 0. Let g^u and g^c denote the equilibrium contribution levels with and without population uncertainty, respectively. Then $g^c >$ (resp. $<, =$) g^u.*

[12] Andreoni (2006) used impure altruism as a synonym of warm glow. Here the term warm glow refers to the direct utility derived from the giving, which is captured by $\psi(\cdot)$.

[13] For example, $\psi(g, n) = gn^\alpha, \alpha \in (0, 1)$ could be one of the functional forms that satisfy the assumptions.

Proof: See Appendix B.

By closing off the channel of responses to the nonlinear production function, we can make direct predictions of the behavior of experiment participants: If their warm glow is increasing and concave in n, then population uncertainty drags down their contribution in the linear VCM. If the warm-glow utility is constant in n, as assumed in the previous section, there will be no significant changes in the contribution level. Of course, an increasing and convex warm-glow utility in n may imply that their contribution will increase with population uncertainty (at least for the group sizes being tested), but this may be counterintuitive, as it implies that when the size of the population gets larger, an individual's contribution increases at a faster rate.

It may be difficult or impossible to tell how people will respond to population uncertainty when both the public-goods production function and the warm-glow utility are increasing and concave. In that case, uniqueness of the equilibrium is not guaranteed, as opposed to the case where the warm-glow utility is independent of n. In addition to the contrasting theoretical predictions, there may be other factors that lead to contrasting directions in the response to population uncertainty. If experiment participants interpret the uncertainty in the population size as ambiguity in the return on their contribution to the production of public goods, they might want to increase the allocation to their private account so that their utility will come from a more certain source. On the other hand, if conservative participants worry more about the worst-case scenario, where the population size turns out to be small, that is, if their risk averseness drives them to put more weight on the possibility that the contributor pool is small, they may want to contribute more. In this case, population uncertainty may still play an important role, even when the mean population size is large, if a potential contributor subjectively perceives that the probability of a small population is substantial. Another possibility is that the salience of population uncertainty will change the subjects' decision process by directly affecting their warm glow. If population uncertainty prompts subjects to give greater consideration to the strategic aspects of the experiment, they may behave more rationally and in a self-interested manner, that is, population uncertainty may drive out warm glow. However, we cannot exclude the possibility that population uncertainty will encourage them to be more altruistic.

The experimental design is described in the following section. Because of limitations on lab capacity and other practical considerations, the experiments have a simpler form than the models discussed in this section, but the fundamental aspects of voluntary contributions

Table 1: Summary of Theoretical Predictions

$u_i(y, g_i, G; n) = (y - g_i) + \psi(g_i, n) + f(G)$, where $G = \sum_{i=1}^{n} g_i$		
$f(G)$	$\partial\psi(g_i, n)/\partial g$	**Comparison**
	0	$\boldsymbol{g^c = g^u}$
kG	Constant in n	$\boldsymbol{g^c = g^u}$
$(k < 1 < nk)$	Concave in n	$\boldsymbol{g^c > g^u}$
	Convex in n	$\boldsymbol{g^c < g^u}$
	0 or Constant	$\boldsymbol{g^c < g^u}$
$k\ln(G)$	Concave in n	**indeterminate**
	Convex in n	$\boldsymbol{g^c < g^u}$

This juxtaposes the theoretical predictions for different production functions with different assumptions about the (marginal) warm-glow utility. Except for the case where the warm-glow utility is concave in n, the individual contribution levels are not smaller under population uncertainty. If the warm-glow utility is concave in n, population uncertainty drags down the individual contribution levels, which renders the comparison between theoretical contribution levels indeterminate when the public-goods production function is $k\ln(G)$.

of public goods that I set out to capture are the same.

4. Experimental Design and Procedures

To test the extent to which voluntary contributions are driven by warm glow versus by the response to population uncertainty, I conducted four sets of experiments, which are summarized in Table 2. The four treatments differed in two dimensions: the functional form of the public-goods production function and whether the group size was certain or uncertain. I abbreviate the treatments as LC (Linear+Certainty), LU (Linear+Uncertainty), NC (Nonlinear+Certainty), and NU (Nonlinear+Uncertainty). LC and LU will be collectively called the linear treatments, and NC and NU will be called the nonlinear treatments. Similarly, LC and NC are called the certainty treatments, and LU and NU are called the uncertainty treatments.

The purpose of the linear-treatment sessions was to investigate how $\psi(g_i, n)$ is affected by population uncertainty. The nonlinear-treatment sessions were performed to examine how subjects respond to population uncertainty. The details follow.

Subjects in the linear treatment sessions played a series of linear voluntary-contribution games: They were endowed with 10 virtual tokens per round, and their task was to allocate the tokens between a private account and a group account in order to earn as much as they

Table 2: Summary of Experimental Design

Treatment	Production Function	Group Size	#Subjects
LC	$4\sum g_i$	6	36
LU	$4\sum g_i$	3 or 9	45
NC	$30\ln(\sum g_i + 1)$	6	30
NU	$30\ln(\sum g_i + 1)$	3 or 9	45

- Each token allocated to the private account had a value of 10 experimental currency units (ECU).
- "3 or 9" denotes that the subjects were told that the group size in each round would be either 3 or 9 with equal probability; the actual group size was revealed after they made their decisions.
- Instead of showing the functional form ($30\ln(\sum g_i + 1)$), a table (printed), a graph (printed), and an interactive graph (on screen) were provided.
- The marginal return on g_i was set to 4 in the linear treatments, so that full contribution was the socially optimal allocation even when the group size was 3.
- The marginal return on g_1 was set to $\frac{30}{\sum g_i + 1}$ in the nonlinear treatments, so that the actual earnings from all four treatments be similar. (In the pilot experiments, subjects allocated about 30%–40% of their endowment into provision of the public good. When every subject contributes 40% of their endowment to the group account, the expected earnings in the LC treatment are the same as in the NC treatment.)

could. Tokens allocated to the private account earned 10 experimental currency units (ECU) apiece. Tokens allocated to the group account earned 4 ECU per token for each member of the group. For example, if a participant allocated eight tokens to the private account and his group allocated a total of 11 tokens (including his two tokens) to the group account, then he would earn 124 ECU (10*8 from the private account plus 4*11 from the group account) for the round. As long as the group size is at least 3, the Pareto-optimal contribution to the group account is 10, while the non-cooperative Nash equilibrium contribution is 0. With this linear public-goods production function, population uncertainty plays no role in and of itself, unless the subjects' warm glow is affected by population uncertainty. The main difference between the LC treatment and the LU treatment is the timing of disclosure of the group size. In each round of the LC treatment, subjects were told at the outset that the group size was 6. In each round of the LU treatment, subjects were told that the group size was either 3 or 9, with a 50% chance for each, but the actual group size was not revealed until after they made their decisions. They were informed that the group size in each round would be randomly drawn by a computer and that it was independent of any history of events.

Nonlinear treatment sessions were conducted similarly, with one difference: The amount of ECU earned by subjects from tokens allocated to the group account was $30 \ln(\sum g_i + 1)$. Instead of a formula or a calculator, a conversion table, a graph, and an interactive graph were provided.[14] Table 3 is part of the conversion table provided to subjects. Based on the total number of tokens allocated to the group account, every member of the group earned the amount of ECU shown in the table, whether or not she had contributed to the group account. For example, if a total of 11 tokens were allocated to the group account, every member earned 75 ECU for the round. If a subject allocated 8 tokens to her private account and her group allocated a total of 11 tokens (including her two tokens) to the group account, she earned 155 ECU ($10 * 8 = 80$ from the private account plus 75 from the group account) for the round. If a group allocated no tokens to the group account, every member earned 0 ECU from the group account.

Table 3: Earnings from the Group Account (Part of the Whole Table)

Tokens in the group account	Earnings (in ECU)	Tokens in the group account	Earnings (in ECU)	Tokens in the group account	Earnings (in ECU)
0	0	8	66	16	85
1	21	9	69	17	87
2	33	10	72	18	88
3	42	11	75	19	90
4	48	12	77	20	91
5	54	13	79	21	93
6	58	14	81	22	94
7	62	15	83	23	95

This table maps the total number of tokens allocated to the group account to the total value of those tokens. For example, if the subjects in a group allocated a total of 11 tokens to the group account, then every member of the group earned 75 ECU.

After subjects read the instructions, they took a quiz to check their understanding of how the experiment worked. For the nonlinear treatments, two of the questions tested whether subjects read the table or the graph correctly. Those who failed to pass the quiz were asked to retake the quiz until they got all the answers correct. After the quiz, they had four rounds of unpaid practice with computer players. They were told that the main purpose of the practice rounds was to become familiar with the interface. They were also told that the computers would make their decisions randomly, so the computer players' decisions might not be similar to those of real participants.

[14]See the Appendix for the table and the graph.

The main experiment ran for 20 rounds. The participants were informed that though the interface for the actual rounds would be the same as the interface for the practice rounds, they would be playing with real participants in the laboratory and the earnings of every round will be paid.

After the main experiment, subjects completed a debriefing survey in which they were asked to indicate their gender and ethnicity, as well as their risk preferences and their level of familiarity with the experiment.

The experimental sessions were conducted at the Experimental Social Science Laboratory (ESSL) at University of California, Irvine from April 24 to May 4, 2017. Python and its application Pygame were used to computerize the games and establish a server–client platform. The participants were UCI undergraduate students, recruited through the ESSL online recruitment system. Sessions ran about 40 to 50 minutes. The session sizes for the certainty treatments were 12, 18, and 24 (multiples of 6), and the sizes for the uncertainty treatments were 18 and 27 (multiples of 9). A total of 156 subjects participated in the experiments.

Participants were randomly assigned to separate desks equipped with a computer interface. They were not allowed to communicate with other participants during the experiment, nor were they allowed to look at other subjects' computer screens. It was also emphasized to participants that their allocation decisions would be linked with the computer number of their seat, so their identity would remain anonymous. Though new groups were formed in every round, there was no physical reallocation of the subjects, and they knew only the size of the group (or, in the uncertainty treatments, only the two possible sizes of the group). The participants input their decisions on their computer interface. The server computer collected all the decisions, formed new groups, calculated the payoffs, and returned each of the payoffs to the corresponding computer.

The participants started out by reading instructions and practicing with a tutorial program. The instructions for all treatments were of similar length and at a similar reading level,[15] and can be found in Appendix A. An instructor answered all questions until every participant thoroughly understood the experiment.

The total amount of ECU each participant earned was converted to US dollars at the rate of 0.0055 USD per ECU.[16] Participants earned \$22.91 on average, including a \$7 participation

[15] The instructions were at approximately an eighth-grade reading level, according to the Flesch–Kincaid readability test, which is a standard measure of how difficult a piece of text is to read.

[16] They were told that the payment would be calculated by the server computer, so they were not to worry

payment.

5. Results

The main results of the lab experiments are summarized as follows:

1. Comparison between NC and NU: The average contribution to the group account was larger when the group size was uncertain than when it was certain.
2. Comparison between LC and LU: No significant difference in the contribution levels was observed. This observation does not support the hypothesis that $\psi(g_i, n)$ is increasing concave in the group size.
3. Gender, ethnicity, and risk preferences were not significant factors.

5.1. Nonlinear Treatments

In this section we focus on the nonlinear treatments, where the public-goods production function was $30 \ln(\sum g_i + 1)$, which is increasing and concave. When the group size was uncertain at the time subjects made their contribution decisions, the average percentage contribution to the group account (27.59%) was slightly larger than the average percentage contribution when the group size was certain (24.03%). For the last 10 rounds, this difference (24.38% vs. 16.97%) is statistically significant. This observation is consistent with the theoretical prediction that subjects respond to the group-size uncertainty. This does not mean, however, that we can explain the contribution levels solely on the basis of group-size uncertainty, without acknowledging the role of the warm-glow utility, $\psi(g_i, n)$: If $\psi(g_i, n) = 0$ for any g_i and n, the Nash equilibrium contribution is 3.333% when the group size is certain at 6, and 7.325% when the group size is uncertain and either 3 or 9. In short, the treatment effect is consistent with the theoretical prediction, but the contribution levels per se would have been much lower if it were not for $\psi(g_i, n)$. Results of the regressions for the nonlinear treatments, including individual characteristics and risk preferences as explanatory variables, are given in Table 4. Since the individual choices are positively correlated across rounds, cluster-robust standard errors were used.

5.2. Linear Treatments

In the LC treatment, where the public-goods production function was linear and the group size was certain, the average contribution to the group account (28.71%) was quite large in

about this conversion.

Figure 2: Average Contribution by Round: Nonlinear Treatments

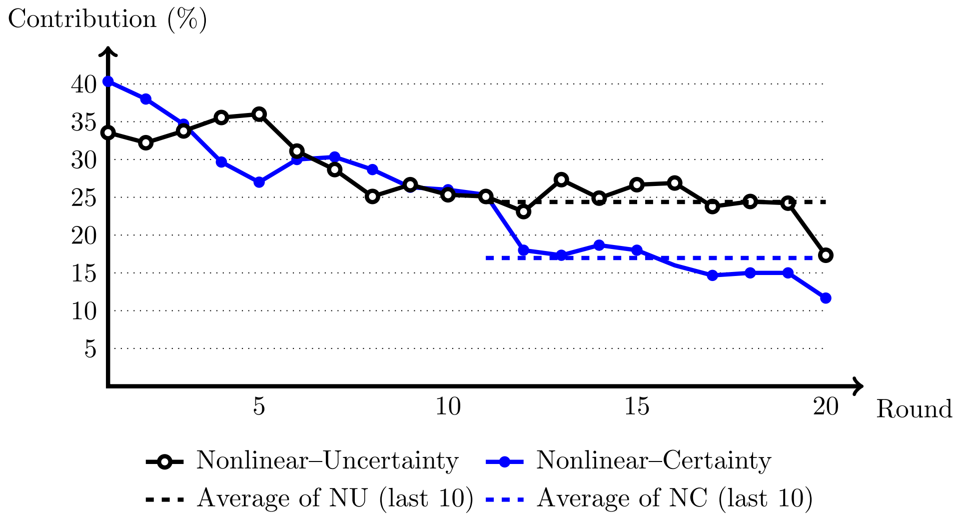


Table 4: Regression Results: Nonlinear Treatments, #Obs. = 1500

Contribution	$\hat{\beta}$	p-value	$\hat{\beta}$	p-value	$\hat{\beta}$	p-value
Constant	3.4058***	< 0.001	3.8391***	< 0.001	3.7797***	< 0.001
Round	-0.0955***	< 0.001	-0.1367***	< 0.001	-0.1367***	< 0.001
Uncertainty	0.3556	0.3771	-0.3667	0.5177	-0.2822	0.6443
Round*Uncertainty			0.0688**	0.0271	0.0688**	0.0271
RiskAversion					0.2164	0.4175
Female					0.2376	0.5437
Asian					-0.4314	0.3377
Adjusted R^2	0.0571		0.0631		0.0778	

A total of 75 subjects participated in the nonlinear-treatment sessions. As predicted by Proposition 1, the average contribution level was larger when the group size was uncertain. In the earlier rounds, the treatment effect was not significant, but as learning took place on the part of the subjects the treatment effect was more distinct. The difference between the average contribution when the group size was certain at 6 and the average contribution when the group size was uncertain and either 3 or 9 is statistically significant at the 5% level. Estimated relative risk aversion, gender, and ethnicity are not factors that drove the contribution level. The regression results are similar when an explanatory variable *After10* is used in lieu of *Round*, where *After10* is a dummy variable indicating whether the current round is after round 10.

comparison to the theoretical prediction of zero contributions. Even for the last 10 rounds, the average contribution (24.75%) did not decrease by much. This was observed in many previous VCM experiments. In the LU treatment, where the group size was uncertain, the average contribution level (33.27%) was larger than that in which there was no uncertainty in the group size. For the last 10 rounds, the average contribution (30.33%) was also larger than the average contribution for the last 10 rounds in the LC treatment. However, those differences were not statistically significant. As the contribution level in the linear treatments was more volatile than that in the nonlinear treatments, the cluster-robust standard errors for the linear treatments were larger than those for the nonlinear treatments. This observation does not support the hypothesis that $\psi(g_i, n)$, the utility term that captures the desire to contribute to the group, is increasing and concave in the size of the group—at least for the sizes considered in this experiment.[17] Nevertheless, it does not mean that my results are contradictory to the findings of Andreoni (2007) or Exley (2016). In contrast to these two studies, where givers do not directly benefit from their contributions, in my experiments, the participants were contributing to the production of public goods, hence the participants were among those who would benefit from their contributions. Results of the regressions for the linear treatments, including individual characteristics and risk preferences as explanatory variables, are given in Table 5.

5.3. Other data

While waiting for the payments to be made, subjects were asked to complete a short survey. I gave them the option of disclosing their gender and ethnicity, as well as the number of recognizable acquaintances in the lab and their level of familiarity with the experiment. The subjects' risk preferences were also surveyed. That was done by using the dynamically optimized sequential experimentation (DOSE) method (Wang et al., 2010), where I asked subjects to answer at most two questions, which enabled me to categorize them into one of seven types in terms of their risk preferences. The second question they were asked was conditional on their response to the first question. (See Figure 4.) Based on the answers

[17]The null effect on population uncertainty under a linear production function could mean (1) $\psi(g, n)$ doesn't depend on n, (2) $\psi(g, n)$ does depend on n, but the functional form is exactly quadratic in n so that $\partial\psi^2(g, n)/\partial n^2$ is 0, or (3) $\psi(g, n)$ cannot be factorized so that the signs of $\frac{\partial\psi^2(g,n)}{\partial n^2}$ and $\frac{\partial\psi^3(g,n)}{\partial g \partial n^2}$ are different for some parameters. If the second interpretation is correct, it means that there is a certain "tipping point": People contribute more as a population size (smaller than the tipping point) increases, but contribute less as a population size (larger than the tipping point) increases. This interpretation contradicts to the evidence that people contribute to the society even in a very large population (Andreoni, 2006). If the third interpretation is correct, we may need to revisit the question studied by Andreoni (2007).

Figure 3: Average Contribution by Round: Linear Treatments

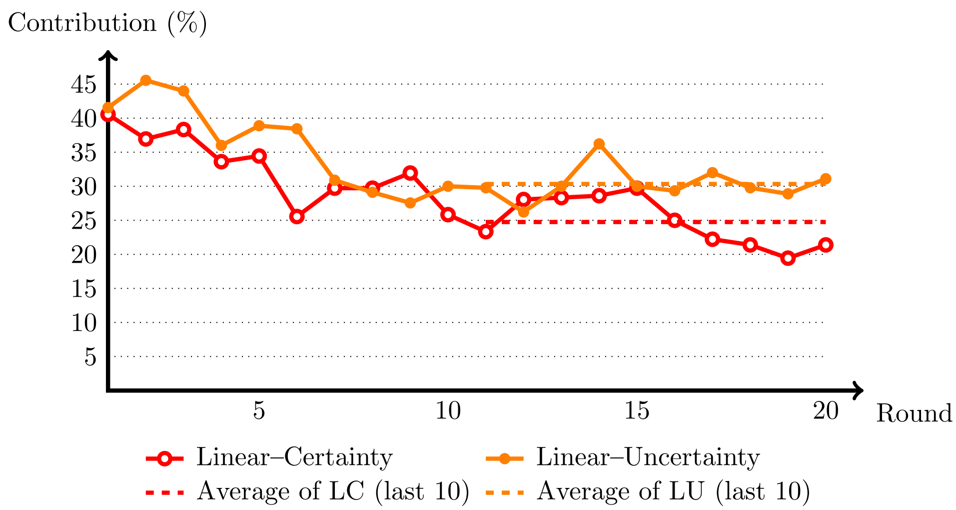


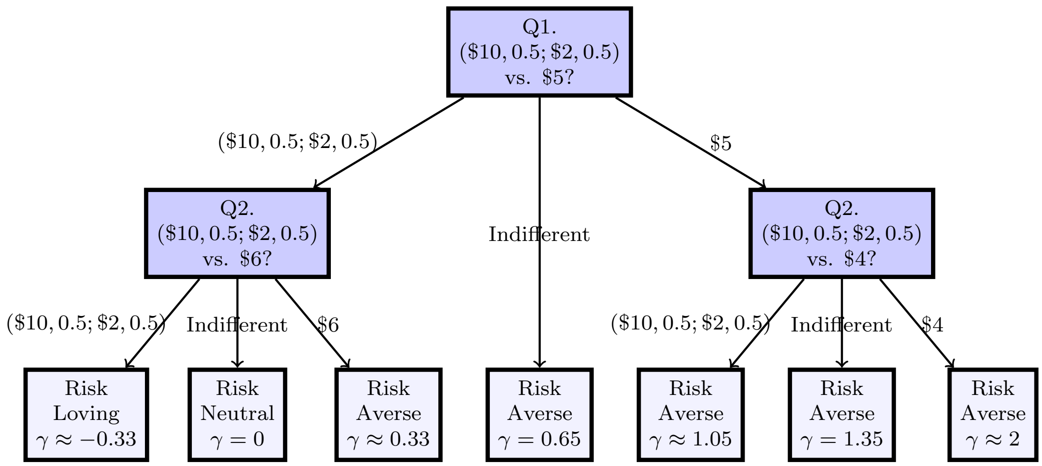
Table 5: Regression Results: Linear Treatments, #Obs. = 1620

Contribution	$\hat{\beta}$	p-value	$\hat{\beta}$	p-value	$\hat{\beta}$	p-value
Constant	3.6682***	< 0.001	3.7860***	< 0.001	4.6820***	< 0.001
Round	-0.0759***	< 0.001	-0.0872***	< 0.001	-0.0872***	< 0.001
Uncertainty	0.4558	0.3700	0.2439	0.6795	0.1324	0.8190
Round*Uncertainty			0.0202	0.5013	0.0202	0.5013
RiskAversion					-0.1876	0.5250
Female					-0.5910	0.2374
Asian					-0.8857*	0.0953
Adjusted R^2	0.0258		0.0258		0.0563	

A total of 81 subjects participated in the linear-treatment sessions. If $\psi(g_i, n)$ is congestible, that is, if it is increasing and concave in n, as assumed in Proposition 2, then the average contribution level should be strictly smaller when the group size is uncertain. The treatment effect was not significant in any of the 20 rounds. Estimated relative risk aversion and gender are not factors that drove the contribution level. Subjects who self-reported as Asian contributed about 0.9 tokens less than those of other ethnicities, which is statistically significant at the 10% level.

to the questions, I estimated the value of each subject's relative risk aversion parameter γ for the case of CRRA utility. Though it is known that risk averseness tends to be smaller without financial incentives (Camerer and Hogarth, 1999), the regression results should be robust if the order of the actual values of the subjects' aversion parameter γ is consistent with that of my estimated values.

Figure 4: Design of Two Survey Questions



The first question is: [If someone gives you a choice between the following two options, which one will you choose? Option A: "I'll give you $10 if a fair coin lands heads, and $2 otherwise." Option B: "I'll give you $5."] If a subject chooses option A, then the second question asks him to compare option A with B′, a more attractive option than B. If a subject chooses option B on the first question, then the second question asks him to compare option A with B′′, a less attractive option than B. If a subject answers the first question by indicating that he/she is indifferent between option A and option B, then the second question is skipped.

No interesting relationships between risk preferences and contribution behavior were observed. Those who self-reported their ethnicity as Asian contributed fewer tokens to the group account in the linear treatments, at the 10% level of significance. There were no significant individual characteristics that drove the main finding in the nonlinear treatments. Comprehensibility of the game affected neither the contribution level nor the final payoffs: I compared those who failed to pass the quiz at least once with those who passed the quiz on their first try, and no interesting differences were observed.

6. Discussions

Among many directions for extension of this study, I discuss the behavior of a large population and dynamic aspects of the contributions in this section.

6.1. Limiting Behavior under Population Uncertainty

Although the model studied in this paper renders some theoretical predictions for a large population, it is still unclear whether the actual contribution levels would increase or decrease with population uncertainty when the mean population is fairly large. There are at least two testable hypotheses for a large population: First, with population uncertainty the contribution level will be less volatile to changes in the mean population size. Second, given the same mean population size, a population distribution which is more skewed to the right (that is, one with a smaller mode) will yield a larger level of contribution, as it emphasizes the possibility that the population size will turn out to be small. The following propositions help to clarify why this would happen. I consider two specific forms of the utility function and each of which respectively corresponds to the model considered in section 3.1 and section 3.2. Specifically, Proposition 3 considers the case where the marginal production of the public goods production function is decreasing and convex while the warm-glow utility does not depend on the population size, and Proposition 4 addresses the case where the production function is linear while the warm-glow utility is increasing concave. To emphasize the expected population size, let $g^u(n)$ and $g^c(n)$ denote the symmetric equilibrium contribution level with and without population uncertainty, respectively, when the expected population is n. When the population uncertainty grows, the free-riding incentive grows more slowly. I use big-$\mathcal{O}$ notation to describe limiting behavior.[18]

Proposition 3. *Suppose $U(x_i, g_i, G) = x_i + a \ln g_i + b \ln G$. Suppose that the number of players is n for $g^c(n)$, and that it is randomly drawn from a discrete uniform distribution $U[2, 2n-2]$ for $g^u(n)$. Then $(g^u(n) - a)/(g^c(n) - a) = \mathcal{O}(\ln(2n))$.*

Proof: See Appendix B.

Proposition 3 states that as n, the mean population size, goes to infinity, both $g^c(n)$ and $g^u(n)$ shrink to a, that is, the voluntary contribution level in equilibrium asymptotically depends only on the warm glow (Andreoni, 2006). However, the rate of convergence to a is $\ln(2n)$ times slower when the population uncertainty grows as the mean population size does. Of course, this rate of convergence depends on the functional form of the utility and the shape of the population distribution. The upshot of this observation is that if the population

[18] Let $f(x)$ and $g(x)$ be two functions on $\mathbb{R}$ and $g(x) \neq 0$ for all x. $f(x)$ is $O(g(x))$ as $x \to \infty$, or $f(x)$ is the same-order relation with $g(x)$ if there exist positive constants M and X such that $|f(x)/g(x)| \leq M$ for all $x \geq X$.

uncertainty gets larger as the expected population grows, players are more reluctant to free ride.

Proposition 4. *Suppose* $U(x_i, g_i, G) = x_i + n^a \ln g_i + bG$, $a \geq 0$, $b \in (0,1)$. *If the number of players is* n, *then* $g^c(n) = \mathcal{O}(n^a)$. *If it is randomly drawn from a discrete uniform distribution* $U[2, 2n-2]$, *then* $g^u(n) = \mathcal{O}(n^{a-1})$ *if* $a < 1$, *and* $\mathcal{O}(n^a)$ *if* $a \geq 1$.

Proof: See Appendix B.

Proposition 4 states that as n goes to infinity, $g^u(n)$ converges while $g^c(n)$ does not when $a < 1$. If $a \in (0,1)$, that is, the warm-glow utility is increasing and concave in n, as assumed, then the equilibrium contribution level diverges without population uncertainty, and converges with population uncertainty. Since charitable giving is stable and consistent in the real world, this may suggest that people who derive utility from warm glow may take population uncertainty into account. If $a \geq 1$, $g^u(n)$ diverges as fast as $g^c(n)$ does, but this case is less relevant to the real world, since $a \geq 1$ implies that the warm glow increases exponentially as the population size grows. Table 1 shows the summary of theoretical predictions. With ignoring the effect of warm glow, the individual contribution levels are higher under population uncertainty. On the other hand, if the warm-glow utility is concave in the population size, it is unpredictable to tell whether population uncertainty boosts up or drags down the individual contribution levels. Thus testing the effect of population uncertainty at a laboratory may be especially important in establishing an empirical basis.

6.2. Resolving/Increasing Uncertainty over Time

The impact of resolving uncertainty over time through recurring interactions could be another important extension of this study. For example, charities often post how many contributors donated in previous years. Potential contributors often take into account other contributors' observable characteristics to make their own contribution decisions. In this paper, I consider a static situation where all agents make their decisions simultaneously and they do not access any of existing histories or previous interaction records.

In theory, resolving population uncertainty over time would *decrease* overall contributions. The main driving force behind the effect of population uncertainty is potential contributors' concern about the possible realization of a small population. In the sense that population uncertainty would be resolved over time only in the direction of increasing the lower bound of the population distribution while keeping the upper bound, it always incurs smaller contributions from the late-coming contributors. To illustrate this, consider the following example. Suppose the population size is either 2 or 10 with equal probability. When all the potential contributors make their contribution decisions simultaneously, they would contribute $g^u(6)$ each. If the population is realized at 10, then the total sum of the contributions will be $10g^u(6)$. If they make decisions at a different time, the players who haven't decided yet would update the population distribution. In the example considered here, when they learn two players made contributions of $g^u(6)$, they immediately know that the population size is 10 for sure, and therefore, instead of contributing $g^u(6)$, they would contribute $g^n(10)$ (or something smaller than $g^n(10)$, with recognizing that $g^u(6)$ is greater than $g^n(10)$). The total sum of the contributions will be at most $2g^u(6) + 8g^n(10)$ in this case. Since $g^n(10) < g^u(10) < g^u(6)$, the overall contributions in the latter case is strictly smaller than the former.[19]

However, it is a hasty conclusion for an institution which relies on voluntary contributions not to resolve population uncertainty, because we do not know whether there are other important factors that have not been accounted for the model. For example, we haven't considered a herding effect and information as a signal. If many potential contributors, instead of contemplating how much contribution they should make, simply follow what many others make, resolving population uncertainty may be beneficial. If the institution hasn't built up their reputation in regards to reliability, revealing the number of previous contributors as well as the contribution amounts would be helpful for potential contributors to update their belief about the institution positively. Therefore, the effect of resolving population uncertainty could be the subject of an empirical research project worth investigating.

7. Conclusions

Population uncertainty has rarely been studied in the literature on voluntary contribution of public goods despite its importance. With population uncertainty added to the decreasing, convex marginal production of public goods, the equilibrium voluntary contribution level is greater than that when the population size is fixed at the mean of the population distribution. With population uncertainty added to the increasing concave warm-glow utility, the voluntary contribution level is smaller than that when the population size is fixed. From the lab experiments, we learned that an individual's voluntary contribution level does respond

[19]This conclusion holds in more general environments. With any population distribution, information about the previous contributors truncates the lower bound of the distribution, and as a result the mean population size of the updated population distribution always increases, and the variance of the updated population distribution always decreases. Both results lead to a smaller contribution level in equilibrium.

to population uncertainty, and we were able to verify many aspects of the theoretical predictions. When the marginal production was decreasing and convex, subjects contributed more under population uncertainty, which is consistent with the theoretical prediction. When the marginal production was constant, there was no significant effect of population uncertainty on the level of contributions. This confirms that, at least for the group sizes considered, the additive utility which captures the social preferences regarding the very act of making a contribution exists and the utility term is independent of the group size. Gender, ethnicity, and risk preferences were not the main factors driving the level of contributions to the production of public goods.

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Appendix A

Sample Experimental Instructions

[Let subjects sign in. Randomly assign subject's seat so that orders in the sign-in sheet do not match the seat number.]

This is an experiment in group decision making. Please pay close attention to the instructions. You may earn a considerable amount of money which will be paid in cash at the end of the experiment. The amount of money you earn will depend on the decisions you make, as well as the decisions other participants make. In addition, you will get a $7 participation fee if you complete the experiment. After the tutorial, you will take a quiz about the experiment. The reason for having a quiz is to make sure that you understand how the experiment works.

The whole experiment consists of 20 rounds of simple tasks whose overall procedure is the same. In each round, you will be asked to make a choice in a slightly different situation and to input your decision to the computer interface.

Your choices and answers will be linked with a computer number of your seat. We will never link names with your responses in any way. Your personal information provided for your payments will never be stored nor used for any research. You will neither learn about other participants' identity in the course of this experiment, nor will they learn about your identity.

You are not allowed to talk to others during the experiments. If you have a question, please raise your hand so that the experimenter can come to you and answer your question in private.

Now please wake up your computer. You will see a welcome screen in a gray window. This is a tutorial which will introduce what you will do during the actual experiments, and make you feel familiar. This tutorial is self-contained, so please read through and follow instructions in the tutorial.

Instructions in the tutorial (Nonlinear-Uncertainty version)

Overview:

The whole experiment consists of 20 rounds. In each round, you will be endowed with 10 virtual tokens. Your task is to allocate the tokens to your private account and a group account. The details follow.

How the groups are formed:

In each round, all participants will be randomly assigned to one of groups. The size of each group will be either three or nine with equal (50%) chance, which is randomly determined by a computer in the beginning of the round. You will make a decision without knowing the actual group size for that round. The actual group size will be revealed after everyone in the session made a decision.

In the next round, the size of the group will be randomly chosen by a computer again. You will be randomly reassigned to a new group. Both the group size and the group assignment will be independent to the previous events. That is, the group size and assignment will never be related to the previous realizations of the group size, your previous actions, and the previous group assignments in any ways.

In any round you will not know who your group members are. Your group members will not know who you are either. Since you are reassigned to a new group of a new group size, it will be unlikely for the members in the previous group to comprise the current group again.

How your earnings are determined:

Every token in the private account has a value of 10 experimental currency units (ECU). You also earn ECU from tokens assigned the group account according to the table and graph given to you. For example, if a total of 11 tokens are allocated to the group account, every member earns 75 ECU from the group account. (Please check it with the table. More details will follow later.)

If you allocate, for example, 8 tokens to your private account and your group allocates a total of 11 tokens (including your two tokens) to the group account, you earn 155 ECU (75 ECU from the group account + 80 ECU from your private account) for the round.

Your total earnings for the experiment will be a sum of earnings in each round. The total amount of ECU you earn will be converted into US dollars at the rate of 0.0055 USD per 1 ECU. For example, if you earn 1000 ECU, you will be paid $5.5, plus $7 of the participation fee. The payment will be calculated by the server computer, so don't worry about this conversion.

What you will know at the end of each round:

After every subject in this session made a decision, the size of the group will be revealed. In addition, you will be informed about (1) how many tokens are allocated to your group account, (2) how many tokens you allocated to your private account, and accordingly, (3) how much you earned for the round. The history of these results will be briefly displayed on the screen.

How to read the table and the graph:

Earnings from the group account are described in the table and the graph given to you. If you want to know how much you will earn from a certain number of tokens in the group account, you may look at the table, find the number on the left column, and check the corresponding earnings in ECU. The graph contains the same information: You may find the number of tokens on the x-axis, and check the corresponding earnings on the y-axis. You can also use an interactive graph on the screen: When you click any area on the graph, it will show the nearest (tokens, earnings)-coordinate. If you need further assistance to read the table or the graph, please ask the experimenter.

Summary of the Process:

1. The experiment consists of 20 rounds of simple tasks.
2. In each round, you are endowed with 10 virtual tokens. Your task is to allocate the tokens to your private account and a group account.
3. At the beginning of each round, all subjects are randomly assigned to one of groups. The size of each group is either three or nine, with equal chance. A computer randomly determines it.
4. After everyone in the session made a decision, the group size, a total number of tokens collected in the group account, and your earnings for that round are revealed.
5. Next round, you repeat this process.

We are now ready to conduct a quiz on these instructions. Are there any questions before the quiz? You may start over the tutorial to review the instructions.

Sample Quiz

1. **(Basic Information)** In each round, how many tokens are you endowed with?
2. **(Basic Information)** Which of the followings is a WRONG statement?

   A The size of each group will be either three or nine, with equal chance.

   B A computer randomly determines the group size in the beginning of the round.

   C The more you allocate tokens in the group account, the group size will be more likely nine.
3. **(Basic Information)** In each round, you will be randomly reassigned to a new group of a new group size. Which of the followings is a CORRECT statement?

   A If your group size was 3 in the current round, the group size of the next round must be 9.

   B It will be unlikely for the whole members in the current group to comprise the next group.

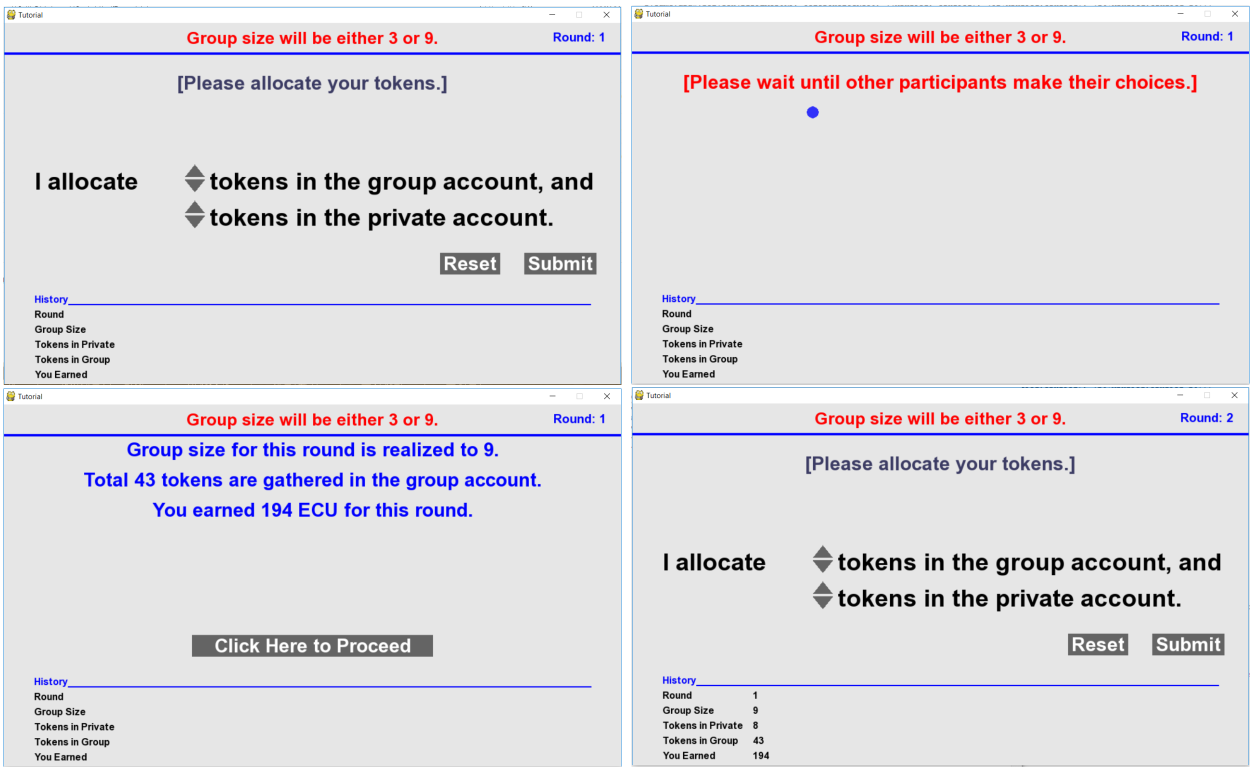
   C At least two group members in the current round will be reassigned to your group.
4. **(How to read the table/graph)** Suppose your group members allocate 20 tokens in the group account together. How many ECU will you earn from the group account?

5. **(How to read the table/graph)** Suppose your group members allocate 7 tokens in the group account together. How many ECU will you earn from the group account?
6. **(Calculation of the payoffs)** Suppose you allocate 8 tokens in your private account, and your group members allocate 11 tokens (including your 2 tokens) in the group account. How many ECU will you earn in total (from both the private account and the group account)?
7. **(Calculation of the payoffs)** Suppose you allocate 6 tokens in your private account, and your group members allocate 19 tokens (including your 4 tokens) in the group account. How many ECU will you earn in total (from both the private account and the group account)?

[Those who pass the quiz do the demo practices.]

Now you will have non-paid practice of four rounds, to make you feel familiar with the interface. In these practice rounds, your group members are computers. Note that the computers are set to choose their decision randomly, so computers decisions may not be similar to that of real participants.

Figure 5: Screen Captures of the Practice Rounds (for the NU treatment)



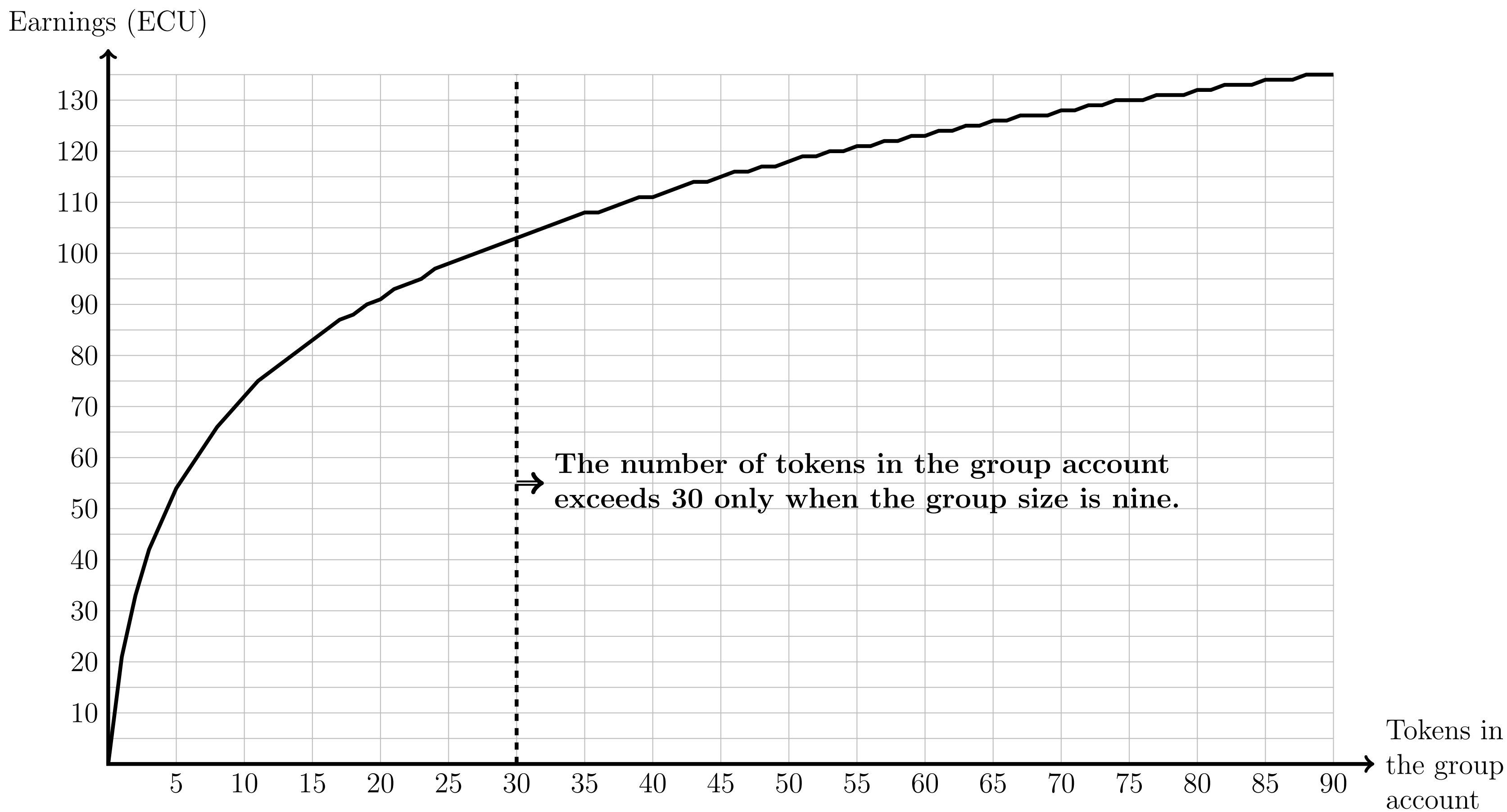
On top of the screen, it is informed that the group size will be either 3 or 9 (top-left figure). After subjects made their decisions, they wait until the last subject made his/her decision (top-right figure). After everyone made a decision, the group size, the number of tokens allocated to the group account, and the earnings for that round are revealed (bottom-left figure). In the next round, the history of previous rounds is displayed on the bottom of the screen (bottom-right figure).

Table given to participants

When the group size is 3		When the group size is 9			
Tokens in the group account	Earnings (in ECU)	Tokens in the group account	Earnings (in ECU)	Tokens in the group account	Earnings (in ECU)
0	0				
1	21	31	104	61	124
2	33	32	105	62	124
3	42	33	106	63	125
4	48	34	107	64	125
5	54	35	108	65	126
6	58	36	108	66	126
7	62	37	109	67	127
8	66	38	110	68	127
9	69	39	111	69	127
10	72	40	111	70	128
11	75	41	112	71	128
12	77	42	113	72	129
13	79	43	114	73	129
14	81	44	114	74	130
15	83	45	115	75	130
16	85	46	116	76	130
17	87	47	116	77	131
18	88	48	117	78	131
19	90	49	117	79	131
20	91	50	118	80	132
21	93	51	119	81	132
22	94	52	119	82	133
23	95	53	120	83	133
24	97	54	120	84	133
25	98	55	121	85	134
26	99	56	121	86	134
27	100	57	122	87	134
28	101	58	122	88	135
29	102	59	123	89	135
30	103	60	123	90	135

Graph given to participants

An interactive graph was also provided on the screen.



Appendix B: Omitted Proofs and Large Population Properties

Proof of Proposition 1 By Jensen's inequality, $\sum_{n=2}^{\infty} \pi(n) f'(ng^c) \geq f'(\sum_{n=2}^{\infty} \pi(n) n g^c) = 1 - \psi'(g^c)$. Since $f'(\cdot)$ is decreasing on account of concavity, g^u such that $\sum_{n=2}^{\infty} \pi(n) f'(ng^u) = 1 - \psi'(g^u)$ has to be at least as great as g^c. For the sake of contradiction, suppose $g^c > g^u$. Since $\psi(\cdot)$ is increasing and concave, $\psi'(g^u) \geq \psi'(g^c)$, or equivalently, $1 - \psi'(g^c) \geq 1 - \psi'(g^u)$. This implies that $\sum_{n=2}^{\infty} \pi(n) f'(ng^c) \geq \sum_{n=2}^{\infty} \pi(n) f'(ng^u)$, and hence $g^u \geq g^c$, in contradiction to the supposition. Note that the equality holds only when $f'(\cdot)$ is linear. $\square$

Proof of Proposition 2 When $f'(\cdot) = k$ and $\partial\phi(g,n)/\partial n > 0$, equation (1) becomes $\sum_{n=2}^{\infty} \pi(n)\phi(g^u, n) + k = 1$. If $\frac{\partial^2 \phi(g,n)}{\partial n^2} < 0$, that is, $\frac{\partial^3 \psi(g,n)}{\partial g \partial n^2} < 0$, by Jensen's inequality $\sum_{n=2}^{\infty} \pi(n)\phi(g', n) < \phi(g', \mu)$ at some g'. The proofs of the other two cases ($\frac{\partial^2 \phi(g,n)}{\partial n^2} > 0$ and $\frac{\partial^2 \phi(g,n)}{\partial n^2} = 0$) are analogous. $\square$

Proof of Proposition 3 To avoid unnecessary algebra, consider instead that the number of players is drawn from a discrete uniform distribution $U[1, 2n-1]$, where the mean of the distribution is the same with that of $U[2, 2n-2]$. This modification does not change the limiting behavior of the rate of convergence, because the probability to draw 1 or $2n-1$ will shrink down to zero. By equation (3), $g^c(n) = a + b/n = \mathcal{O}(1)$, and $\lim_{n\to\infty} g^c(n) = a$. By equation (2), $g^u(n) = a + b/n \sum_{i=1}^{2n-1} 1/i$. Since $\sum_{i=1}^{2n-1} 1/i$ is a left Riemann sum of $1/x$ from 1 to $2n$ and $1/x$ is decreasing in x, $\sum_{i=1}^{2n-1} 1/i \geq \int_1^{2n} (1/x) dx = \ln(2n)$. Similarly, $\sum_{i=1}^{2n-1} 1/(i+1)$ is a right Riemann sum of $1/x$ from 1 to $2n$, hence $\sum_{i=1}^{2n-1} 1/(i+1) \leq \ln(2n)$, or $\sum_{i=1}^{2n-1} 1/i \leq \ln(2n) + 1 - 1/(2n)$. Thus $a + (b/n)\ln(2n) \leq g^u(n) \leq a + (b/n)(\ln(2n) + 1 - 1/(2n))$. Now $0 < 1 - 1/(2n) < \ln(2n)$ and $\lim_{n\to\infty}(1/n) ln(2n) = 0$. Therefore, $g^u(n) = \mathcal{O}(1)$, $lim_{n\to\infty} g^u(n) = a$, and $(g^u(n) - a)/(g^c(n) - a) = \mathcal{O}(ln(2n))$. $\square$

Proof of Proposition 4 By equation (3), $g^c(n) = n^a/(1-b) = \mathcal{O}(n^a)$. Consider again that the number of players is drawn from a discrete uniform distribution $U[1, 2n-1]$. By equation (2), $g^u(n) = \sum_{i=1}^{2n-1} i^a/((2n-1)(1-b))$. When $a < 1$, $g^u(n)$ is $\mathcal{O}(n^{a-1})$, since the largest term in the summation is $(2n-1)^a/((2n-1)(1-b)) = (2n-1)^{a-1}/(1-b)$. All the other smaller terms shrink to zero. The k-th largest term $(2n-k)^a/((2n-1)(1-b)) = \{(2n-k)/(2n-1)\}^a (2n-1)^{a-1}/(1-b)$ converges to zero since $\{(2n-k)/(2n-1)\}^a$ converges to 1 and $(2n-1)^{a-1}$ converges to 0 as n goes to infinity, for all $k \in \{2, 3, \ldots, 2n-1\}$. When $a > 1$, however, $(2n-1)^{a-1}$ diverges so each term in the summation diverges at the rate of $(2n-1)^{a-1}$. Since the number of terms is $2n-1$, $g^u(n) \leq (2n-1)(2n-1)^a/(2n-1)/(1-b) =$

$(2n-1)^a/(1-b)$, hence $g^u(n) = \mathcal{O}(n^a)$. $\square$