Penalty Lottery*

Duk Gyoo Kim†

November 28, 2022

Abstract

To control sequential public bad productions under imperfect monitoring, this paper proposes a penalty lottery: A violator passes the responsibility of the fine to the next potential violator with some probability and pays all the accumulated fines with the complementary probability. The penalty lottery does not merely impose extreme fines because an absorbing state is practically unreachable. It *self-selects* people more willing to produce public bads and endogenously imposes the larger expected fines on them. It has advantages over the day-fine system in which the fine depends on the offender's daily income. Experimental evidence is consistent with the proposed theoretical predictions.

Keywords: Institutional change, Public bads, Misdemeanors, Imperfect monitoring, Laboratory experiments

1 Introduction

Tickets issued for misdemeanors are a multi-billion industry. More than ten million citations for parking violations were issued during the fiscal year 2017 in New York City.[1] The minimum fine is $35, so the parking tickets alone are at least a 378-million-dollar revenue generator. If that number represents the social costs incurred by the negative

*I thank Andrzej Baranski, Antoine Camous, Yongmin Chen, Hans Peter Grüner, Randi Hjalmarsson, Harim Kim, Sang-Hyun Kim, Jinhyuk Lee, Joosung Lee, Zvika Neeman, Nicola Persico, Euncheol Shin, Kathryn E. Spier, Thomas Tröger, Leeat Yariv, Sangjun Ye, Minchul Yum, Galina Zudenkova, seminar participants at the University of Mannheim, Yonsei University, Korea University, Seoul National University, and Sungkyunkwan University, and conference participants in the 35th meeting of the European Association of Law and Economics, the 13th Nordic Conference on Behavioral and Experimental Economics, the 2019 Asia-Pacific Meeting of the Economic Science Association, HeiKaMaX Workshop, and Reading Experimental and Behavioral Economics Workshop for their helpful comments. The replication package, including the entire dataset and the experiment instructions and screenshots, is available in the Open Science Foundation repository q7ykx.

†Department of Economics, Sungkyunkwan University, *kim.dukgyoo@skku.edu*

[1] Source: NYC Open Data

externalities of wrongdoing, it is natural to consider the proper way to minimize misdemeanors. A day-fine system, in which a unit of fine payment is based on the offender's daily personal income, is employed in some countries, and it has been considered an alternative to fixed fines (Hillsman, 1990). Several news media outlets have covered the day-fine system in Finland.[2] However, it is still debatable whether the wealth level—a factor not directly related to the offending action—justifies the different fines for the same wrongdoing and whether it would prevent the misdemeanors of the poor.[3] Spending more resources to enhance public monitoring, such as hiring more police officers, is not always implementable. Increasing the fine may not be justifiable for multiple reasons. If a fine is precisely determined at the level to correct the negative externality of wrongdoing (Becker, 1968), there is no ground for increasing the fine. Even so, if a local government enforces exceptionally severe penalties to eradicate all incidences of public bads, this leads to distorted incentives on the margin (Stigler, 1970). If, for example, every misdemeanor results in a substantial fine, so the punishment for littering is almost the same as for burglary, then a person willing to litter might also be willing to break into a structure. As discussed later, another potential issue of the unilateral fine increase is that it deters the actions of citizens with low willingness to produce public bads, while citizens with high willingness remain undeterred.

Given that the fundamental reason for prohibiting a misdemeanor is that it produces a public bad, a broader goal is to seek efficient ways to reduce an individual's sequential public bad production under imperfect monitoring. Although the motivating example is parking violations, any problem of bad uses of common resources is also relevant to the broader goal. The primary purpose of this paper is to propose a simple institutional change that helps to attain the goal, which, herein, is called a *penalty lottery*. A violator, a citizen who produced a public bad for their own sake and was monitored, passes over and accumulates the fine for the next violator with some probability q, and that person pays all of the accumulated fines with probability $1-q$. The standard rule of the game is nested as a special case with $q=0$. This institutional change involves neither a change in the public monitoring capacity nor an increase in the nominal fine, but it could reduce the number of public bad producers in the long run. Even in a situation where every individual citizen initially finds public bad production beneficial, this penalty lot-

[2]Daley, Suzanne. April 25, 2015. Speeding in Finland Can Cost a Fortune, if You Already Have One. *New York Times*. Accessed: March 9, 2019.
Pinsker, Joe. March 12, 2015. Finland, Home of the $103,000 Speeding Ticket. *The Atlantic*. Accessed: March 9, 2019.

[3]Another practical issue is how the income level can be transparently measured: The fact that a day-fine system is currently employed in countries where the degree of public tax transparency is high suggests that establishing a high level of public tax transparency may be the first-order requirement for implementing a day-fine system.

tery asymptotically prevents every citizen from producing a public bad after some finite period. However, the existence of an 'absorbing state,' where a sufficiently large number of fines are accumulated so that no one violates the law, is neither the main advantage of the penalty lottery nor the central claim raised in this paper. Although the prediction of asymptotic convergence to the absorbing state is robust to (bounded) heterogeneities in citizens' risk preferences and social preferences, it can take substantially longer periods to reach such an absorbing state when a minuscule fraction of the population brings an additional heterogeneity. When the distribution of the heterogeneities is unbounded above, then the steady state is never reached. Hence, this paper focuses more on the short-run dynamics.

The merit of the penalty lottery is that it dynamically *self-selects* those who are more likely to produce a public bad, so it *endogenously* imposes the larger expected fines to them, even without further controls from the government. If people subjectively evaluate the fine relative to their income, then the penalty lottery could endogenously build up the day-fine system. If individuals' risk preferences are different, risk seekers are willing to produce a public bad when the size of accumulated fines is larger than what the risk-averse can afford, making them share the burden of the large accumulated fines. This implies that when the penalty lottery is introduced, the expected benefit of risk seekers might not be larger than that of the risk-averse.

This paper claims that the penalty lottery, or the random pass of the penalty, would work in various situations in which the primary challenge is to minimize the production of public bads with imperfect monitoring capacity, such as reducing carbon dioxide emissions of firms and controlling individual spending of the common goods.[4] In particular, this institution would work as a mechanism to offer different fines based on the offender's willingness to commit misdemeanors. A citizen decides whether to violate the law for his own sake, knowing that his action bears a small chance of paying all the accumulated fines. It does not mean that the system unilaterally imposes unjustly-added penalties on every citizen. Those who find the accumulated fines large will discontinue committing misdemeanors and therefore have to pay no penalties. Only those who are willing to take the risk of paying all the accumulated fines will commit misdemeanors. In this regard, the penalty lottery has significant advantages over the day-fine system. Under the penalty lottery, a unit of (ex-ante) fine payment is based on the offender's willingness to commit misdemeanors, so it endogenously implements the day-fine system currently employed in some developed countries. At best, the day-fine system can be justified when

[4]Under *joint and several liability* in tort claims, a plaintiff may recover all the damages from any of the defendants, regardless of their share of the liability. In the sense that the winner of the penalty lottery takes the whole accumulated fines, the idea is understood as one particular rule of joint and several liability.

the personal daily income works as a proxy for how strongly the offender minds paying the fine. The penalty lottery makes them directly reveal their true willingness.

Testing this new institution and citizens' responses to it in real-world jurisdictions is challenging, especially when we do not know whether there are unexpected adverse outcomes from the institutional change. In particular, although joint and several liability in tort claims has some related features, behavioral responses to the payment of the other's fines have not been reported, so we are unsure how people would behave under this new institution. This is why I propose using laboratory experiments as a testbed.

I consider two treatments in each of them differs in the probability ($q \in \{0.1, 0.5\}$) of accumulating the fine rather than paying it. The treatment with $q = 0.1$ works as a baseline experiment, which mimics the current rule of the game. During the experiment, one subject per each round either chooses a red ball (corresponding to a law-abiding action) or a blue ball (a violation of a law), knowing how many black stickers (accumulated fines) are in the common pool that the group members share. When the subject chooses a blue ball, a black sticker is attached to the blue ball with a 30% chance. That is, the probability of his action being monitored is 0.3. In the case of drawing a blue ball with a black sticker, the black sticker is accumulated to the common pool with probability q. However, with the other probability $1-q$, he will keep the black sticker and all the stickers in the common pool. Subjects get paid based on the number of red balls, blue balls, and black stickers that they have kept during the experiment. After the main experiment and the post-experiment survey, the subjects' risk preferences were measured by the Bomb Risk Elicitation Task (Crosetto and Filippin, 2013).

The experimental evidence is largely consistent with the theoretical predictions. First, as more black stickers (fines) are accumulated in the common pool, fewer subjects chose the blue ball (violation). Although the treatment with $q = 0.5$ gives more incentives for them to choose the blue ball as the joint probability of getting the stickers is low, the accumulation of the black stickers leads them to choose the red ball (law-abiding action) about two times more frequently than those in the treatment with $q = 0.1$.[5] Second, risk-seeking behavior does not pay better under the treatment with $q = 0.5$. When $q = 0.1$, taking the risk of getting the black stickers is beneficial, so there is a significant negative relationship between risk aversion and earnings in the main experiment. When $q = 0.5$, however, the negative relationship is completely nullified.

The rest of the paper is organized as follows. In the following subsection, we discuss the related literature. Section 2 describes the model and provides the main theoretical implications. Section 3 describes the experiments, and Section 4 reports experimental

[5]This result communicates to Drago et al. (2009), who examine the behavioral responses of (exogenously determined) heavier penalties to recidivism by using a natural field experiment.

findings. Section 5 discusses various issues, including comparisons with other mechanisms, the use of information technologies to ensure common knowledge, and the objective of local government, and Section 6 concludes. Proofs of the lemmas and propositions are provided in Appendix A, and the sample instructions for the experiment are in Appendix B.

1.1 Related Literature

This study is inspired by Gerardi et al. (2016), Duffy and Matros (2014), and Bhattacharya et al. (2014), who study how an institutional change affects voter turnout. Gerardi et al. (2016) compare a lottery (giving a prize to one voter of the turnout) with a fine (to those who do not turn out to vote), and Duffy and Matros (2014) consider a combined mechanism in which the fines imposed on non-participants are used to finance the prize of the lottery winner. In a similar vein, Bhattacharya et al. (2014) compare compulsory voting with voluntary voting. In a sense that we examine how an institutional change drives citizens to behave more desirably, this paper contributes to the literature on the impact of institutional changes.

This study is also related to imperfect public monitoring of the behavior of producing public bads. Ambrus and Greiner (2012) report how imperfect monitoring, compared with perfect monitoring, changes the contribution behavior in public good contribution games with a costly punishment option. Their experimental evidence shows that access to a standard punishment technology significantly decreases net payoffs even in the long run, and access to a severe punishment technology leads to roughly the same payoffs as with no punishment option. This finding implies that it is hard to make individuals act in a socially desirable way, even if a severe punishment is adopted. The main advantage of the penalty lottery over a simple severe punishment is that it works well under imperfect monitoring.

Using lotteries in a non-standard setting is not a new idea. Kearney et al. (2010) overview prize-linked savings (PLS) accounts, which essentially provide 'no-lose' lottery tickets to savers, and Filiz-Ozbay et al. (2015) examine the validity of PLS using lab experiments.[6] Morgan (2000) theoretically shows that funding public goods utilizing lotteries outperforms voluntary contributions in that lotteries increase the provision of public good close to the first-best level, and the theoretical predictions are consistent with the experimental findings of Morgan and Sefton (2000). Kim (2021) provides an experimental evidence that a lottery for encouraging vaccination works better than a small

[6]Texas Proposition 7: Financial Institutions to Offer Prizes to Promote Savings Amendment was approved on November 7, 2017. This proposition allows banks and credit unions in Texas to offer PLS accounts as a savings option.

lump-sum transfer, especially to probability-weighting subjects. The penalty lottery may seem as odd as it gets, but turnout lotteries, voluntary contribution lotteries, and savings lotteries had also been odd until they were explored in academia.

2 Theoretical Framework and Implications

Although the model can be applied to general situations of sequential public bad production under imperfect monitoring for punishment, we consider a problem of conducting a misdemeanor for clearer illustration. Violating the law for one's own sake can be read as producing a public bad.

Consider a society with a continuum of citizens indexed by i. A random citizen sequentially[7] faces a problem of wrongdoing for his/her own benefit at a discrete time $t \in \mathbb{N}_+$.[8] For terminological clarity, I call those who violate the law as "wrongdoers" and some of the wrongdoers who are caught by a police officer as "violators." Let B_i denote the units of benefit from wrongdoing, C_i denote the units of cost to abide by the law, and F denote the amount of the fine. To make the problem nontrivial, assume that $F > 0$, $B_i > C_i > 0$, and $(1-p)u_i(B_i)+pu_i(B_i-F) > u_i(-C_i)$, where $p \in (0,1)$ is the common probability of being monitored (hence ticketed),[9] and $u_i(\cdot)$ is an increasing concave utility function of citizen i. That is, the expected utility of wrongdoing is greater than that of paying the cost of abiding by the law for all i. In this scenario, every citizen is better off by wrongdoing.

Now I consider one additional state variable. A violator is asked to pay the fine with probability $1-q$. With probability q, the violator passes the responsibility of the fine to the next violator, and the fine is accumulated to a public account. Let F_t denote the accumulated fines at time t, with $F_0 = 0$. That is, at time t, a citizen faces a problem of choosing either B_i or C_i while knowing that the probability of paying the fine, $F_t + F$, is $p(1-q)$. The timing of the event is described in Figure 1. Note that the typical rule of the game is nested in this institution as a special case with $q = 0$. I assume here that the local government's goal is to minimize infractions and misdemeanors.[10]

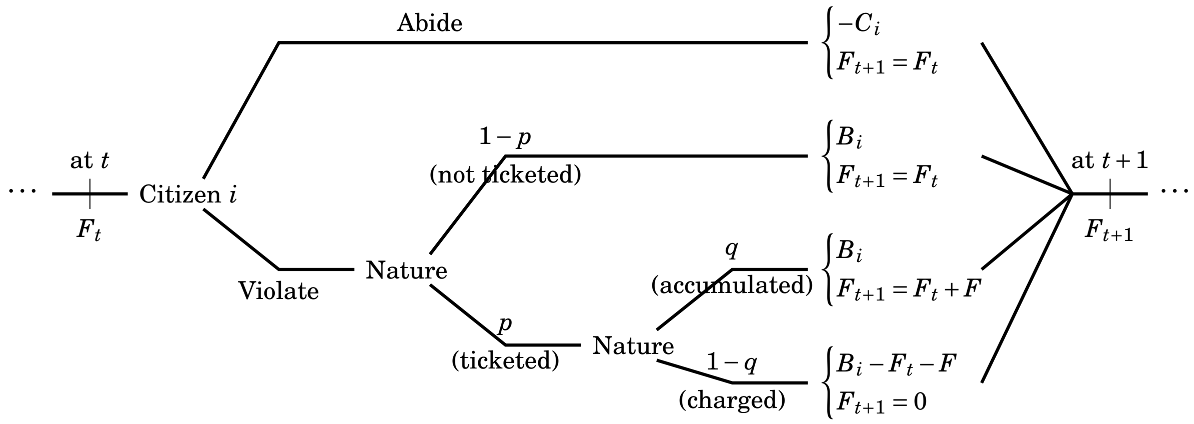
[7]Their decisions are independent of what others are doing. This sequential arrival can also be interpreted that each citizen does not encounter others in a situation where wrongdoing is considered.

[8]Alternatively, we can consider that each of n citizens sequentially arrives at $t = (\tau-1)n + i, \tau \in \mathbb{N}_+$, where the finite number of citizens and the circular time index are used for lab experiments. It does not mean that the model should assume that every citizen decides on a circular queue.

[9]It is worth noting that the subjective belief about such a probability may vary by individual and change over time. Hjalmarsson (2008) reports that the perceived punishment severity discontinuously increases with the age of the criminal majority. However, the model predictions are largely unaffected when we replace the common probability p with a subjective probability p_i.

[10]Considering that the state court system's budget typically relies on funding from traffic citations as a source of revenue, this may not be a trivial assumption. This is discussed further in Section 5.

Figure 1: Timing of events



For any $q \in [0,1)$, a finite number of citizens would violate the law as the probability to pay the fine, $p(1-q)$, is lower than p. However, once the fines are sufficiently accumulated, the expected benefit of wrongdoing is no longer greater than the cost of abiding by the law. Specifically, for any τ such that $(1-p+pq)u_i(B_i)+p(1-q)u_i(B-(F_\tau+F)) \leq u_i(-C)$, rational citizen i would not violate the law. For risk-neutral citizen i, there exists $k_i \in \mathbb{N}_+$ such that $k_i = \left\lceil \frac{B_i+C_i}{p(1-q)F} \right\rceil - 1$, where $\lceil x \rceil$ is the smallest integer larger than x, if there are k_i consecutive violators who have not paid the fines, citizen i will not violate the law afterwards. Since k_i is a cutoff level of fine accumulation for a risk-neutral citizen, k_i can be understood as the upper bound[11] of cutoffs for any degree of risk aversion. Although this model deals with three types of heterogeneities, all the other various dimensions of individual heterogeneities can also be summarized by k_i, and k_i is interpreted as a degree of willingness to violate the law. Let $\bar{k}$ denote $\max_i k_i$, and $G(k)$ denote the cumulative distribution of k_i. The actual number of wrongdoers varies by how many violators happened to pay the fines before reaching $F_t = \bar{k}F$. However, the probability of $\bar{k}$ consecutive accumulations approaches one as time goes by. Let $P(V|t;p)$ be the ex-ante probability that a violation is observed after t periods when the probability being monitored is $p \in (0,1)$. Let $P(V|t)$ denote $P(V|t;p)$ whenever p is not of interest.

Proposition 1. *For $q \in (0,1)$, $P(V|t)$ is monotone decreasing in t. The expected time to reach $P(V|t) = 0$ is finite. If $q = 0$, $P(V|t) = 1$ for any t.*

Proof: See Appendix.

[11]The assumption of weak concavity of the utility function applies here. If some citizens have a convex utility function, which implies that they enjoy taking risks, then they may have a higher cutoff threshold than k_i.

Proposition 1 becomes intuitive if we consider a homogeneous economy where $u_i = u$, $B_i = B$, and $C_i = C$ for all i. Then $P(V|t;p)$ approximates the probability that a success run of at least length k for an event with a probability of q occurs at least once within pt[12] number of trials. The nondecreasing probability of success run in the number of trials implies that for a longer period, it becomes more likely to reach the threshold kF, and no one will violate the law with a higher probability. For example, if $\{B,C,F,p,q\} = \{5,1,8,0.4,0.5\}$, $k = 3 (= \lceil \frac{5+1}{0.4 \cdot 0.5 \cdot 8} \rceil - 1)$, and the probability of at least 3 consecutive accumulations in 100 periods is 95.66%. Such a probability monotonically increases in t.

In sum, after the society consecutively accumulates k fines, it reaches a 'steady state' where no one violates the law. This result is *not* the main theoretical insight. Although we ascertained that the penalty lottery may lead to the complete elimination of sequential public bad production in the long run, it is based on the bounded heterogeneities of the citizens' characteristics. When individuals are more heterogeneous, for example, even when a minuscule fraction of the population with $\bar{k}+1$ added to the society, a substantial (in the magnitude of power of $\bar{k}+1$) delay in reaching the steady state is expected. Simply put, the steady state would not practically be reached. Proposition 2 summarizes this claim.

Proposition 2. *The expected number of time periods reaching the steady state starting from zero fine accumulations is*

$$\frac{1}{pq} \sum_{i=0}^{\bar{k}-1} \frac{1}{(1-G(i))q^i},$$

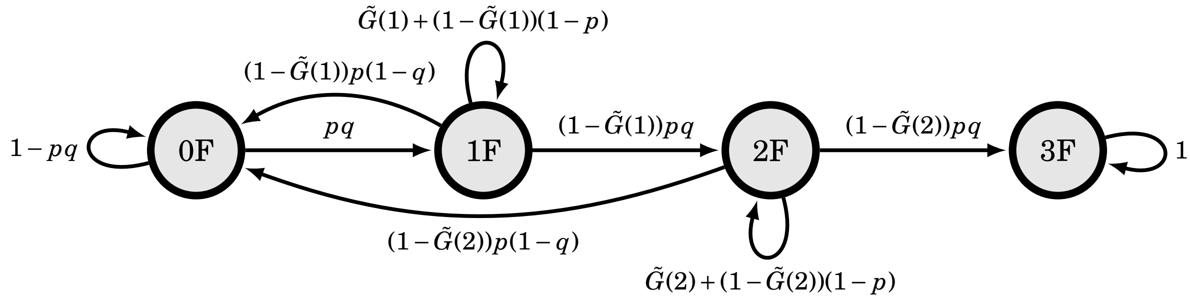
and it implies that if $\tilde{G}(k)$ is the mean-preserving spread of $G(k)$ with a larger $\bar{k}$, and $\tilde{P}(V|t)$ is the ex-ante probability under $\tilde{G}(k)$, $\tilde{P}(V|t) \geq P(V|t)$.

Proof: See Appendix.

For an illustration of Proposition 2, consider a society where half of the citizens have $k_l = 1$, and the other half have $k_h = 3$. Then, until k_l fines are consecutively accumulated, everyone will violate the law, and for any given time t, pt violations on average will be monitored. However, when k_l fines are consecutively accumulated, only half of the population would still violate the law, and for any given additional time τ, $\frac{1}{2}p\tau$ violations on average will be monitored. Two additional fines need to be consecutively accumulated with a smaller number ($\frac{1}{2}p\tau$ instead of $p\tau$) of probabilistic trials, so it must take

[12]Since we consider imperfect monitoring and only p proportion of the whole wrongdoings are monitored, only tp periods out of t can be considered as Bernoulli trials, on average.

Figure 2: Finite-State Absorbing Markov Chain and Transition Probability Matrices



$$T_G = \begin{bmatrix} 1-pq & pq & 0 \\ p(1-q) & 1-p & pq \\ 0 & 0 & 1 \end{bmatrix} \qquad T_{\tilde{G}} = \begin{bmatrix} 1-pq & pq & 0 & 0 \\ (p-pq)/2 & (2-p)/2 & pq/2 & 0 \\ (p-pq)/2 & 0 & (2-p)/2 & pq/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Consider $\tilde{G}(k)$, a mean-preserving spread of a degenerate distribution at 2, where $g(1) = g(3) = 0.5$. The probability of reaching the final node from the initial node after t periods, $(T^t_{\tilde{G}})_{1,k_h+1}$, is smaller than $(T^t_G)_{1,k+1}$.

a longer time to reach the steady state, compared with a homogeneous society where $k_i = 2$ for all i. Worse yet, if someone with $k_h = 3$ pays all the accumulated fines before reaching $F_t = k_h F$, then the entire population starts violating the law again. Figure 2 illustrates the Markov chain and the corresponding transition probability matrices for the aforementioned example. The probability of observing a violation after t periods is the first element of the last column in T^t, where T is the transition probability matrix. Although the above example considered drastically different distributions, a minuscule change can lead to a significant delay. When the society consists of homogeneous citizens with $k_i = 2$, the probability that a violation is observed after 100 periods is 0.32% (with $p = 0.3$, $q = 0.5$), while if 1% of the population has $k_i = 3$ instead, such a probability skyrockets to 88.95%. More interestingly, compared with a society with 99% of $k_i = 2$ citizens and 1% of $k_i = 3$ citizens, if 99.9% of citizens are of $k_i = 2$ and 0.1% of them are of $k_i = 3$, such a probability is 98.81%, even larger than 88.95%. This illustration implies that even when the society's average willingness to produce public bads decreases (from 2.01 to 2.001), the time reaching the absorbing state can be longer.

A naturally followed question is whether the penalty lottery is desirable for substantively longer before-the-steady-state periods. The answer is yes, as it *endogenously* imposes the larger expected fines to those who are more likely to violate the law.

Corollary 1. *Let $\phi(k_i)$ be the ex-ante expected fine for an individual with k_i. $\phi(k_i)$ is increasing.*

Proof: $\phi(k_i) = \sum_{\kappa=0}^{k_i} p(\kappa)(\kappa+1)$ is the ex-ante expected fine for an individual with k_i, where $p(\kappa)$ is the ex-ante probability that κ fines have been accumulated. □

This result implies that the penalty lottery induces an endogenous price discrimination: The more willing a citizen is to violate the law, the larger expected fines.[13] Among the various dimensions of individual heterogeneities leading to different k_i, I focus on a few crucial dimensions: risk aversions, heterogeneous benefits, B_i, and some social preferences. Corollary 1 implies that those who have more benefits from wrongdoing and those who are more risk-seeking *self-select* to pay the higher ex-ante expected fines. The risk-averse citizens have a small k_i: As the fines are accumulated more, the possible payoffs become more volatile. Thus, their expected benefit of wrongdoing decreases with risk aversion. A similar logic applies to those who have a larger B_i. As the fines are accumulated more, a smaller fraction of citizens with higher B_i would remain wrongdoers. Interpreting $\frac{B_i}{F}$ as a subjective value of the fine,[14] the higher ex-ante expected fine for those with a higher B_i would naturally implement a day-fine system. Lying-averse citizens or sincere citizens can be understood as those with $B_i = 0$. Therefore, lying-averse citizens would not be wrongdoers no matter what q is, and the institutional change does not affect their decisions. If citizens have heterogeneous other-regarding preferences, especially inequity aversions, then those with a higher inequity aversion would have a lower k_i. When $F_t > 0$, a wrongdoer is voluntarily taking the possibility of paying previous violators' fines, which exacerbate the inequity between him and the previous violators (who committed the same infraction). All in all, the effect of the institutional change would be significant and is robust to heterogeneities in individual characteristics.

3 Experimental Design and Procedure

To the best of my knowledge, behavioral responses to the (potential) payment of the other's penalties have not been reported, so it is unclear whether the institutional change would work in the desired direction. This is why controlled lab experiments are used as a testbed.

Having abstract framing[15] in mind, I consider two experimental treatments that dif-

[13] On the contrary, if the penalty size is unilaterally increased to deter wrongdoing, it would work as adverse price discrimination. Those with a little willingness to violate the law are deterred more strongly than those with a high willingness.

[14] More often than not, the benefits of wrongdoing are not measurable, while the fine, especially if it is a monetary penalty, is. From the perspective of the rich, F could be relatively minor compared with their wealth. In this case, a higher $\frac{B_i}{F}$ could mean that citizen i subjectively undervalues the fine.

[15] Although the design of experiments to test the effect of the institutional change might be straightforward, one of the critical challenges in this particular context is to maintain abstract framing. I avoid

Table 1: Experimental Design and Summary of Hypotheses

Treatment	p	q	$p(1-q)$	k	$P(\mathrm{V}\|t=60)$
Mq	0.3	0.5	0.15	4	56.19%
Lq	0.3	0.1	0.27	2	85.44%

Each session consists of 60 consecutive rounds. In each round, one subject among a group of four chooses either a red ball or a blue ball. A red ball has a value of -1. A blue ball has a value of 5, but with probability p, it entails a black sticker whose value is -8. The black sticker is collected to the common pool with probability q.

fer in the probability of penalty accumulation (q). For notational simplicity, the treatment with $q = 0.1$ is called Lq (Low q), and the other treatment with $q = 0.5$ is called Mq (Middle q). Table 1 summarizes the design of the experiments.

The basic procedure of an experimental session is as follows: Each subject is endowed with 100 tokens (the experimental currency units) in his/her account. Each subject is randomly grouped with three other subjects and sequentially decides within the group. The experiment consists of 60 decision-making rounds, so each subject made 15 decisions.[16] When a subject is on her turn, she is asked to choose either a red ball or a blue ball, while knowing how many black stickers are accumulated in the common pool shared by the group members. In the beginning, there are no black stickers in the common pool. Keeping a red ball ends her turn. When choosing a blue ball, with a probability of 0.3, a black sticker is attached to the blue ball, and with the complementary probability of 0.7, no black sticker is followed. If a black sticker is attached, the computer determines where the black sticker goes. With probability q, the black sticker is removed from the blue ball and added to the common pool. With probability $1-q$, the subject keeps the black sticker as well as all the black stickers from the common pool. In each turn, a subject does not know which balls other subjects have chosen but knows how many black stickers are accumulated in the common pool. At the end of the session, the total payoff of a subject who has x red balls, y blue balls, and z black stickers is $100-x+5y-8z$. That is, each red ball (the cost of abiding by the law) is worth -1 token, each blue ball (the

using terms such as "wrongdoing," "violations," "fines," and "misdemeanors": Otherwise, interpretations of the experimental results could be confounded as there could exist an experimenter-demand effect (Zizzo, 2010). It is also known that subjects are heterogeneous regarding internalizing the social norms (Kimbrough and Vostroknutov, 2016), so the statistical analysis would have less power to conclude when the particular frame of violating a law is considered.

[16] There are two reasons why a subject makes 15 separate decisions rather than 15 consecutive decisions as a single player. First, if 15 consecutive decisions were to be made, then the state variable of the next round (the number of black stickers in the common pool) is endogenously affected by the previous decision while having three decisions between a single subject's actions significantly dilutes such endogeneity. Second, although it would be ideal if the size of the group is larger, considering the typical capacity of a laboratory, the group size of four was practically optimal as it is large enough for diluting endogeneity as well as small enough for collecting many observations within a limited time.

benefits of wrongdoing) is worth +5 tokens, and each black sticker (the fine) is worth −8 tokens.

Two treatments that I do not conduct may be worth mentioning. First, when a subject makes a decision, how other subjects had acted is not known to her. Although it is likely that knowing other subjects' decisions would affect the subject to some degree and it would be interesting to know to what extent an indirect social pressure impacts, the primary purpose of the experiments is not on examining such an impact. Second, instead of having $q = 0$, I consider treatments with $q = 0.1$ as a baseline experiment. Several studies including Tversky and Kahneman (1981) and Martínez-Marquina et al. (2019) have reported that an individual's behavior under zero probability (or certainty) is distinctively different from that under near-zero probability (or near-certainty), and it is also known that people tend to treat complex lotteries differently than simple lotteries, even when those two lotteries are equivalent (Huck and Weizsäcker, 1999). Thus, it is asserted that individual decisions on probabilistic events should be compared with those on other probabilistic events, not deterministic events.[17]

Experiments were conducted at the Mannheim Laboratory for Experimental Economics (mLab) from February to April 2018. Three sessions were run for each treatment, with 20 to 24 subjects (or 5 to 6 independent groups of four) each. A total of 128 subjects participated in one of the sessions. The participants were drawn from the mLab subject pool. Python and its application Pygame were used to computerize the games and to establish a server–client platform. All experimental sessions were organized along with the same procedure: After the subjects were randomly assigned to separate desks equipped with a computer interface, they were asked to read the instructions (see Appendix B) of the experiment carefully. After the experimenter read the instructions out loud, the subjects spent time reviewing the instructions and took a quiz to prove their understanding of the experiment. Those who failed the quiz were asked to read the instructions and retake the quiz until they passed. An instructor answered all questions until every participant thoroughly understood the experiment. Although each of the subjects belonged to a group of four, there was no physical reallocation of the subjects, and they only knew that they were randomly grouped. They were not allowed to communicate with other participants during the experiment. It was also emphasized to participants that their decisions would be anonymous, and that no deception is employed. At the end of the main experiment, they were asked to fill out a post-experiment survey and participate

[17]Given that q must not be zero in the baseline, another practical reason for considering $q = 0.1$, not lower than 0.1, is that subjects may find the joint probability to be more complicated than the Mq treatment. During the pilot session, $q = 0.05$ was used, and many subjects struggled to instantly figure out the joint probability $p(1-q) = 0.285$.

in another paid task without detailed instructions.[18]

At the end of the post-experiment survey, the subjects' risk preferences were measured by an additionally paid Bomb Risk Elicitation Task (BRET) (Crosetto and Filippin, 2013). BRET is known to be a parsimonious but accurate way to elicit one's risk preference: One box among 36 boxes[19] hides a bomb, and a subject collects as many as boxes she wants without knowing where the bomb is hidden. If the bomb was in one of the boxes collected, she earns zero. If the bomb was not in the boxes collected, she earns €0.11 for each chosen box. The total amount of tokens that each subject earned during the main session was converted into Euro at the rate of €0.08/token.[20] Payments (€11.94 on average, including the earnings from BRET) were made in private at the end of the session, and subjects were asked not to share their payment information. Each session lasted approximately 1 hour.

4 Experimental Results

To summarize the results, the experimental evidence is largely consistent with the theoretical predictions.[21] First, the proportion of wrongdoers approaches 0 as the size of the accumulated fines increases. Second, risk-seekers are not better off than the risk-averse agents in the Mq treatment, while risk-seekers are better off in the Lq treatment.

Table 2: Data Summary

Treatment	Number of Subjects	Red Balls (%)	Blue Balls (%)		
			w/o Sticker	Common	Subject
Mq	64	18.33	58.75	10.21	12.71
Lq	64	8.85	64.17	3.02	23.96

Table 2 summarizes the data. A total of 128 subjects (64 in each treatment) participated in one of the sessions. Red Balls refer to the percentage of the actual choices of red balls. The number of black stickers in the common pool affects subjects' choices, which will be examined shortly. There are three cases when a subject chose a blue ball. Blue

[18] The instructions of BRET were provided at the end of the post-experiment survey.

[19] Crosetto and Filippin (2013) use 100 boxes, but this study used fewer boxes to shorten the time that the participants spent after the main task was done. Although more boxes with a smaller per-box payoff would provide finer elicitation of the risk preferences, the number of boxes does not affect the risk preference per se.

[20] Subjects were instructed that the currency exchange should not be a concern as the server computer would handle it. The exchange rate of €0.08/token is chosen to make the average earning close to the desired size of payments.

[21] The replication package, including the entire dataset, is available in the Open Science Foundation repository q7ykx.

Balls w/o Sticker refers to the case where the subject chose a blue ball, and it came without a black sticker. Blue Balls Common refers to the case where the subject chose a blue ball, it came with a black sticker, and the sticker is added to the common pool. Blue Balls Subject refers to the case where the subject chose a blue ball, it came with a black sticker, and the sticker, as well as all the stickers in the common pool, are added to the subject's account. The results are well randomized. In the Lq treatment, for example, given that a blue ball was chosen for 91.15% of the whole decision-making rounds, the fraction of those three cases would be close to 63.81% (=0.7*91.15), 2.73% (=0.3*91.15*0.1), and 24.61% (=0.3*91.15*0.9), respectively. Also, it seems that abstract framing worked properly: For 96.25% of all decision-making rounds in the Mq treatment, choosing a blue ball maximizes the subject's (risk-neutral) expected payoff, and in 83.12% of such rounds the blue ball was chosen. Similarly, among 96.88% of the decision-making rounds in the Lq treatment, the blue ball was chosen in 93.01% of such rounds. If subjects viewed the problems faced during the experiment as a situation of violating the law for the sake of their benefits, the fraction of the blue ball choices might have been much lower.

4.1 Responses to Penalty Accumulations

One desirable observation is that subjects respond to the number of black stickers in the common pool. Figure 3 shows how many red balls were chosen for different numbers of black stickers in the common pool. In both treatments, it is upward sloping, which implies that the more fines are accumulated, the fewer subjects choose to produce a public bad. In the Mq treatment, if the number of accumulated black stickers in the common pool exceeds 4, it is indeed not beneficial for risk-neutral subjects to choose a blue ball. Except for one subject who consistently chose a blue ball when five stickers are in the common pool and another subject who did so in the final round, everyone else chose a red ball. In the Lq treatment, the maximum number of stickers in the common pool is 2, which is consistent with the theoretical prediction. Unlike realistic situations, the experiment has a last round, and subjects may choose a riskier option in their last round. Such a "last round effect" is statistically insignificant (see the blue dashed lines in Figure 3).

Table 3 summarizes some of the regression results. Throughout all of the regression models being considered, three explanatory variables are statistically significant at the 1% level: the number of black stickers in the common pool (ComBlack), the subject's previous action of choosing a red ball (prevRed), and the decision rounds (DRound). Consistent with the theoretical prediction and the trend shown in Figure 3, subjects actively respond to the number of black stickers in the common pool after controlling for other explanations. Controlling for other factors, the subjects tend to choose a red ball in later

Figure 3: Proportions of Red Ball Choices

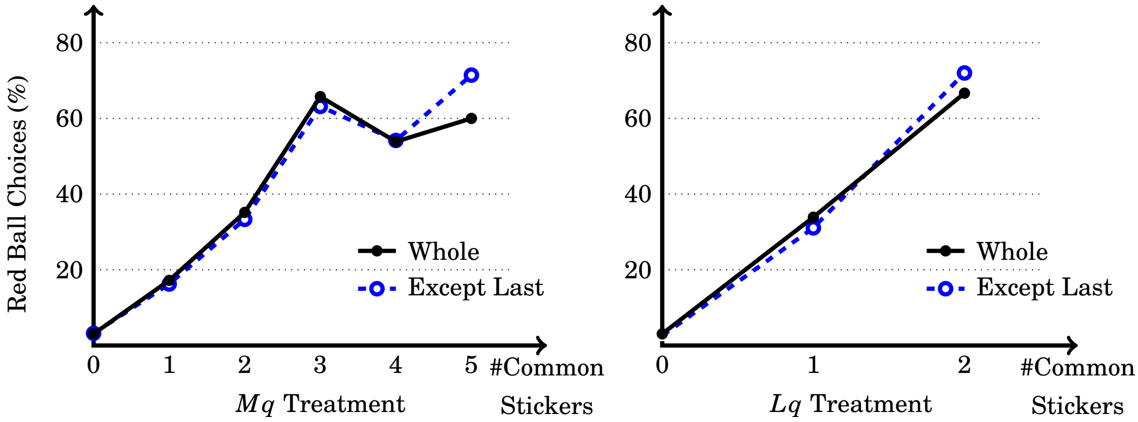


Table 3: Red Ball Choices

	LPM			Logit	
	(1)	(2)	(3)	(4)	(5)
ComBlack	0.2961	0.2953	0.2253	2.2648	1.9263
	(0.0413)	(0.0421)	(0.0403)	(0.3253)	(0.3344)
SubjBlack	−0.0182	−0.0188	−0.0127	−0.1557	−0.1072
	(0.0086)	(0.0091)	(0.0072)	(0.0834)	(0.0717)
Mq	0.0012	0.0051	−0.0008	0.5629	0.4694
	(0.0258)	(0.0261)	(0.0185)	(0.4520)	(0.3502)
ComBlack×Mq	−0.1459	−0.1463	−0.1039	−1.3319	−1.1249
	(0.0466)	(0.0476)	(0.0414)	(0.3829)	(0.3612)
DRound	0.0113	0.0115	0.0080	0.1142	0.0730
	(0.0028)	(0.0029)	(0.0023)	(0.0271)	(0.0246)
prevRed			0.3238		1.8969
			(0.0597)		(0.3535)
prevBCom			0.0325		0.4166
			(0.0403)		(0.3087)
prevBSubj			0.0036		−0.3458
			(0.0133)		(0.3596)
cons.	−0.0190	−0.1970	−0.1066	−5.9808	−5.0881
	(0.0148)	(0.1559)	(0.1095)	(1.7613)	(1.3167)
Indiv.Chars.	Excluded	Included	Included	Included	Included
R^2	0.2711	0.2866	0.3651	0.3036	0.3657
N	1,920	1,920	1,664	1,920	1,664

The dependent variable is the choice of a red ball. ComBlack is the number of black stickers in the common pool, and SubjBlack is the number of black stickers that the subject has kept. Mq is a binary variable indicating whether the treatment of the session was Mq. DRound is the decision round varying from 1 to 15. prevRed, prevBCom, and prevBSubj are binary variables respectively indicating whether the subject chose a red ball in the previous turn, whether she chose a blue ball followed by a black sticker added to the common pool, and whether she chose a blue ball followed by a sticker and kept all the stickers including ones in the common pool, respectively. Standard errors clustered at the subject level are in parenthesis.

decision rounds, and the subjects are more likely to choose a red ball once they chose it in the previous round. It is also reasonable that subjects respond more vigilantly under the Lq treatment, which is captured by negative estimates of the coefficient of ComBlack×Mq,[22] where Mq is a binary variable indicating whether the treatment of the session was Mq. This is because k, the theoretical threshold for a risk-neutral agent to start choosing red balls, is 4 in the Mq treatment, while it is 2 in the Lq treatment. The positive relationship between a previous red ball choice and a current red ball choice implies that some subjects consistently chose red balls: Perhaps they are more risk-averse, more reluctant to take any action that has the potential to negatively impact the other members' payoffs, or more inequity averse.

The previous realization of a probabilistic event also affects subjects' decisions in the following round, especially when the realization is a loss, as many previous experimental studies regarding decision making under uncertainty report so (Imas, 2016). When a subject chose a blue ball in the previous round, and it led him to keep the black stickers, including ones in the common pool (labeled as prevBSubj in the regression table), he tends to choose a blue ball again in the following round. However, this risk-taking behavior after a loss is not as significant as the effect of the number of black stickers in the common pool. Also, some individual characteristics collected by the after-experiment survey—gender, age, self-confidence in their performance of the experiment, and familiarity with this type of experiment—were insignificant.

It is worth mentioning that an income effect would be a valid concern. Since in every decision round, the participants could see the accumulated numbers of red balls, blue balls, and black stickers and how each item is worth (see the screenshots in the online appendix), it is possible for them to accurately calculate the accumulated earnings. Also, since the initial endowment of 100 tokens may work as a reference for determining gains and losses, the subjects' decisions might be different if the accumulated earnings are below or above 100 tokens. To check these potential income effects, I added a dummy variable indicating whether the current earning level is below 100 tokens to the regression models (1)–(3) of red ball choices in Table 3. The estimates of the dummy variable are statistically insignificant in the three models. Regressions with another dummy variable, indicating whether the accumulated earning level is between 95 and 99 (so that the current decision could allow them to earn 100 or more) and yet another dummy variable indicating the level between 101 and 105 (so that their decision could result in them earning 100 or less) show that the red ball choices are not affected. Finally, the effect of

[22]Although all the estimated coefficients of ComBlack×Mq are unilaterally negative, the interpretation should be only based on the linear probability models, as it is known that interaction terms in a Logit model do not have a clean interpretation (Ai and Norton, 2003).

the accumulated earning level is not significant either.[23]

4.2 Risk Seeking does not Pay

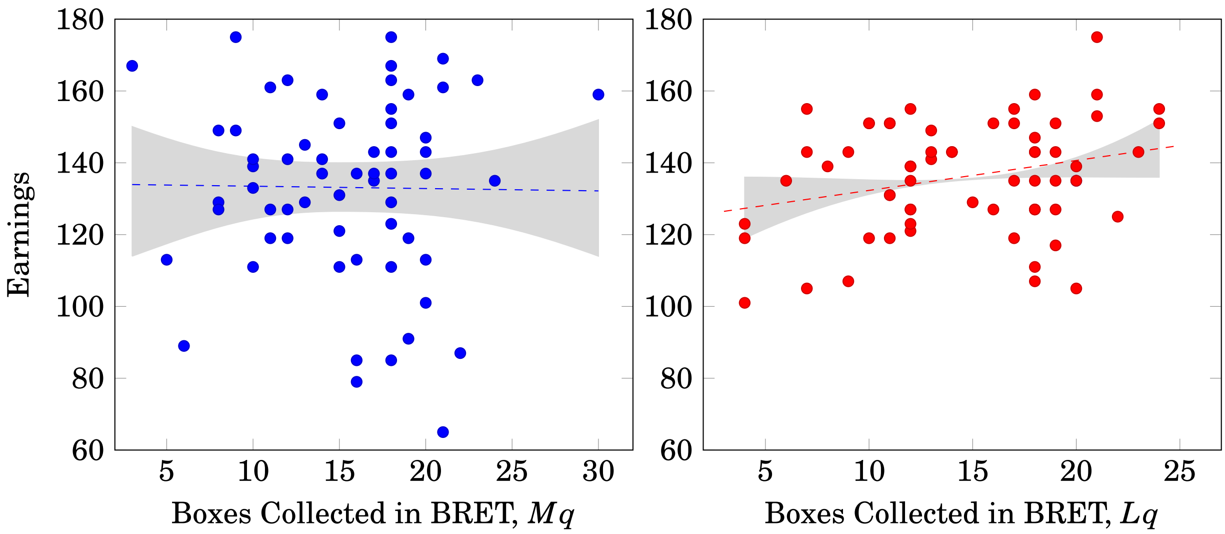
Another crucial theoretical prediction is regarding self-selection: Those who are more risk-seeking are self-selecting to pay the higher ex-ante expected fines. This evidence implies that risk-taking actions will be paid less under the penalty lottery. The experimental evidence supports this claim. Figure 4 shows scatter plots and fitted lines of earnings (from the main experiment) with respect to the number of boxes collected in BRET. A subject's risk preference is elicited by the number of boxes collected in BRET: As she collects more boxes, both the probability that the bomb is included in one of the boxes collected and the payoff when the bomb is not included in one of them increase. Thus, a natural interpretation is that those who collect more boxes in BRET are more risk seeking. In the Lq treatment, those who are more risk-seeking are paid more: Those who collect one more box in BRET earn, on average, 0.83 tokens more ($p = 0.032$). In the Mq treatment, however, there is no linear relationship ($p = 0.918$) between earnings and the number of boxes collected in BRET. The two-sample Kolmogorov-Smirnov test does not reject the hypothesis that the distributions of the two sample datasets on the boxes collected are different ($D = 0.1235$, $p = 0.737$), so it can be regarded that risk-taking behaviors per se are similar in both treatments.

This result, the null relationship between risk preferences and earnings under the penalty lottery, must require careful interpretation. Choosing a risky option (a blue ball) can increase the expected payoff when the expected loss (the number of black stickers in the common pool) is not substantial. That is, even with the penalty lottery, some risk-takers would still be better off than the risk-averse. In this regard, a more accurate but verbose title for this subsection should be "risk-seeking pays less than what it used to pay without the penalty lottery." What we should look into further is the variance of the earnings. Table 4 shows the F-test results of the equality of variances. In the Mq treatment, the variance of the earnings of the subjects whose risk preferences are in the top half is significantly larger than the variance of those in the bottom half. In the Lq treatment, such a difference in variances is not found. This result demonstrates how the penalty lottery works in a way to prevent risk-takers from earning more: Although it is, in general, correct that a high risk yields a high return, with the penalty lottery, a high risk brings higher volatility in payoffs.

One caveat in interpreting the relationship between the earnings from the main experiment and the risk preferences elicited from BRET is that the decisions in BRET are

[23]The unreported regression results are available in the Open Science Foundation repository q7ykx.

Figure 4: Risk Preferences and Earnings



This figure shows scatter plots and fitted lines of earnings with respect to the number of boxes collected in BRET. In the Mq treatment, there is no linear relationship between earnings and the number of boxes collected in BRET. In the Lq treatment, however, those who are more risk-seeking are paid more.

Table 4: F-test of Equality of Variances

Mq		Lq	
St.dev.(Earnings $\mid B_i < \text{Med}(B_j)$)	St.dev.(Earnings $\mid B_i > \text{Med}(B_j)$)	St.dev.(Earnings $\mid B_i < \text{Med}(B_j)$)	St.dev.(Earnings $\mid B_i > \text{Med}(B_j)$)
19.49	28.33	15.22	16.60
$F_{29,28}(2.1117) = 0.0256$		$F_{31,29}(1.1888) = 0.3211$	

assumed to be unaffected by the previous earnings. It should be admitted that there might be an earning effect on the risk preferences, but I claim that there are at least two reasons to believe that such an earning effect is weak. First, the earnings are more dispersed in the Mq treatment, so if there were significant earning effects, then we could have observed a stronger positive relationship between the earnings and the risk-seeking behaviors, which is the opposite of what the data shows. Second, no interim income effects on the decisions in the main treatment (Section 4.1) suggest that the current income level does not affect the decision in the subsequent task. Since the post-experiment survey and the instructions for BRET were between the main experiment and BRET, the time gap between the end of the main experiment and the beginning of BRET was substantially longer than the time gap between the previous decision and the current decision in the main experiment, which could attenuate the earning effect even further.

5 Discussions

5.1 Common knowledge of the accumulated fines

One maintained assumption which puts forward theoretical analysis is that the number of accumulated fines, F_t, is common knowledge. By harnessing existing information technologies, it is not practically hard to provide such information. For example, electronic signboards on highways display varied information, such as the number of car accidents of the previous day in the neighboring area and the dynamic price of road usage (Cramton et al., 2018), which change over time. Adding the current number of accumulated fines to a digital display is not a practical challenge. Of course, for a smaller-scale problem of bad uses of common resources, providing common knowledge must be easier.

Harnessing technologies to design better policies is not a new idea. Cramton et al. (2019) claim to consider efficient pricing of road usage that dynamically depends on the time and the location, while arguing that advances in mobile communications make it possible to access a precise measurement of road use and real-time traffic information. Shoup (2017) recommends that cities should charge fair market prices for on-street parking that dynamically respond to demands. The successful outcomes from several reforms, including the most notable ongoing experiment in San Francisco called SFPark, show that the parking reforms are indeed practical and realistic. If it is practical and realistic to become informed about the dynamic pricing of the parking meter, then it is also practical to inform oneself about the accumulated fines.

Nonetheless, when the costs of providing common knowledge to a large population are not negligible, the penalty lottery can still be used for small-scale public bad minimization. For example, when roommates share a kitchen, informing every roommate of the accumulated costs to clean the kitchen mess so far must not be costly. A sign in front of a public restroom informing people of extra maintenance costs due to abuses of many random passersby should convey necessary information at no cost. My claim is that the costs of providing common knowledge, if unavoidable, are a determinant to the size of the penalty lottery mechanism, not the reason for rejecting this idea.

5.2 Budget constraint of the poor violator

The budget constraint of a violator is not considered in this paper. Provided that the concerns for common knowledge are cleared, the budget-constrained agent will decide not to produce a public bad from the point it is not worth doing so. Therefore, more accumulated fines do not affect the budget-constrained agent at all. Perhaps the following analogy is more intuitive: A longer prison sentence for a felony does not affect ordinary people's ordinary lives as they do not commit a crime. Analogously, someone who decides not to violate a law would not worry about receiving a larger fine that they cannot afford, because it is a worry for naught.

When it comes to an unfortunate nature of a pure mistake, no penalty system can be free from such a concern. Some citizens could not afford a fixed but large fine. Under the day-fine system, a pure mistake of a billionaire comes with a huge price tag. Does it mean that the cost of a mistake should be different based on the daily income level? Even worse, a fired CEO with an enormous previous-year income but no current income might find the fine unaffordable if the 'daily income' is determined according to the tax declaration of the previous year. The concerns about the discrepancy between theory and practice are duly taken, but the possibility of mistakes due to cognitive limitation or inattention cannot be the reason to stop discussing the idea, as this is not a particular problem of the penalty lottery per se.

5.3 Minimization of distorted incentives on the margin

Although I have claimed *neither* that complete deterrence is the main advantage of the penalty lottery *nor* that the absorbing state is easily reached, some may want to know why this mechanism would be considered rather than imposing a massive fine for complete deterrence. The primary reason is that distorted incentives on the margin (Stigler, 1970) are much smaller under the penalty lottery.

Suppose a situation where a fine changes from F to kF with $k \in (1, \bar{k}]$. Those who have $k_i \leq 1$ will not produce a public bad, and those with $k_i > k$ will produce a public bad regardless of the changes in the size of the penalty. Those with $k \in (1, \bar{k}]$ are "on the margin" because they are deterred due to the change. That is, for a significant fraction $(G(k)-G(1))$ of the citizens, their incentives are distorted toward producing more public bads. Under the penalty lottery, in each period with k accumulations, a smaller fraction $(g(k))$ of the citizens have distorted incentives on the margin.

Another practical issue is that the policymaker typically does not know how large the fine should be for it to be sufficient. Even if the policymaker could gauge $\bar{k}$, the smallest size of the fine for complete deterrence, that size does not guarantee the effectiveness in the long run: Perhaps inflation might not be accounted for appropriately, or new citizens might increase heterogeneities. Under the penalty lottery, the policymaker neither needs to know $\bar{k}$ nor needs to track the changes in $\bar{k}$.

The uncertainty about $\bar{k}$ may lead to another potential problem. The unilateral fine increase, if not sufficient for complete deterrence, would deter the actions of citizens with low willingness to produce public bads, while citizens with high willingness remain undeterred. Put differently, relatively "better" citizens' actions are distorted more. When multiplying the fine by k, those with $k_i \leq k$ will earn $u_i(-C_i)$, while those with $k_i > k$ will earn the expected utility from producing public bads, which is greater than their $u_i(-C_i)$.

5.4 The purpose of the local government

Although this paper claims that the penalty lottery would work in general situations of sequential public bad production under imperfect monitoring for punishment, it should be admitted that reducing misdemeanors is one of the most relevant cases. If we narrow down our focus to the elimination of misdemeanors, the primary objective of the local government would matter because it affects the size of the fine and, therefore, the validity of the penalty lottery.

The primary purpose of this study is not to investigate the optimal size of the fine. I assume that a policymaker deliberately measures the negative externality of wrongdoing, such that the fine is set at an appropriate level (Becker, 1968). That is, I assume that the fine is equal to a Pigouvian tax to correct the negative externality incurred by the wrongdoing[24] plus the cost of enforcement execution. If the local government has a balanced budget on misdemeanors, having no revenues with few violations, in the end, is as good as having full (budget balanced) revenues with everyone's violation. It is a plus that in the course of attaining the steady state of no violations, the revenues are almost the same while imposing larger fines to those who are more willing to violate the law.

[24]In this regard of interpreting the fines as indirect taxes, it makes more sense that speeding tickets are more frequently issued to drivers who are not local constituents, as Makowsky and Stratmann (2009) find: This is a way to increase tax revenues from the outside effectively.

However, from many anecdotal pieces of evidence and some rigorous reports,[25] we know that the fundamental goal of the municipalities may not be on eliminating infractions and misdemeanors: Speeding and parking tickets to drivers could be considered an essential revenue source for the local government. If the government's purpose is not to maximize the social welfare, but to maximize the revenue accrued from the fines, minimizing misdemeanors is not in their interest at all. In this case, still, we can exploit the idea of the penalty lottery by introducing a nonlinear penalty accumulation. Since q can be dependent on the current level of accumulated fines, the policymaker of the local municipality may want to set $q(F)$ to maximize the tax revenue from the fines.

A novel type of time inconsistency problem may arise under the penalty lottery if the government maximizes the tax revenue from the fines. Since the citizens self-select the expected fines, the government will gradually know the type of each citizen better. In particular, when a citizen produces a public bad and gets monitored when F_τ is accumulated, the lower bound of their willingness to produce a public bad must be greater than or equal to F_τ. Then, the government may want to use this information to design a "personalized" penalty system based on the revealed willingness in later periods. However, this personalized penalty may be problematic in the sense that this system does not charge the same fine for those with the same willingness to produce a public bad.

5.5 Sincere vs. rational non-violators

Thus far, it has been assumed that $B_i > C_i > 0$ and $(1-p)u_i(B_i)+pu_i(B_i-F) \geq u_i(-C_i)$ for all i, to start from a benchmark case where everyone is better off by producing a public bad. If we relax these assumptions, there could be some citizens who will not produce a public bad under the standard rule of the game. In the context of conducting misdemeanors, I call citizen i with $u_i(B_i) \leq u_i(-C_i)$ as a *sincere* non-violator because he does not violate the law regardless of p, and citizen j whose utility function satisfies $u_j(B_j) > u_j(-C_j)$ and $(1-p)u_j(B_j)+pu_j(B_j-F) \leq u_j(-C_j)$ as a *rational* non-violator. That is, while the penalty lottery does not give an incentive to sincere non-violators to violate the law, it does give an incentive to some rational non-violators if q is too large. For a rational non-violator j, there exists $q_j \in (0,1)$ such that for all $q \geq q_j$, $(1-p+pq)u_j(B_j)+p(1-q)u_j(B_j-F) \geq u_j(-C_j)$.

Even when a large fraction of the population consists of rational non-violators, the penalty lottery will work unless q is not overly high. Consider a 'marginal' citizen among

[25] Garrett and Wagner (2009) find that significantly more tickets are issued in the year following a decline in revenue, but an increase in revenue does not lead to the smaller issuance of tickets. Makowsky and Stratmann (2011) also find that budgetary shortfalls lead to the more frequent issuance of tickets to drivers.

rational non-violators such that $(1-p)u_j(B_j)+pu_j(B_j-F)=u_j(-C_j)$. This citizen is called marginal because he is indifferent between violating and not violating the law when $q=0$, but for all $q>0$, he violates the law if $F_t=0$. Even if the marginal citizen is risk-neutral, he will again abide by the law immediately after $\lceil q/(1-q)\rceil$ fines are accumulated. Thus, if $q \leq 0.5$, then all the rational non-violators will not violate the law when one fine is accumulated in the public account.

Also, I claim that it is not a bad idea to distinguish sincere non-violators from those who strategically choose not to violate the law. Indeed, their violations can be increased without the introduction of the penalty lottery: For example, if the monitoring capacity were to be $p(1-q)$ instead of p, they would have violated the law.

6 Concluding Remarks

This work proposes using lotteries to control public bad production under imperfect monitoring for punishment. The fundamental idea is to randomly accumulate penalties to a public account, rather than to ask each violator to pay the penalty. The penalty lottery has many advantages over both the fixed fine system and the day-fine system. It would not practically reach the steady state of complete deterrence so the theoretical existence of the steady state is neither the main advantage of the penalty lottery nor the main claim of this paper. The penalty lottery is desirable because it endogenously imposes larger expected fines on those who are more willing to produce a public bad. If the primary reason for endorsing the day-fine system is that it exogenously assigns a larger fine for those who might have a higher willingness to produce public bads, the penalty lottery has a significant advantage over the day-fine system because it endogenously induces the day-fine system. This is particularly important if people believe the current system, often cynically described as "evil pays better," needs to be changed. The experimental evidence broadly supports the theoretical predictions. In particular, evil does not pay better in the sense that producing more public bads does not yield larger payoffs.

Although efforts have been made to discuss some realistic issues, it is admitted that this theoretical and experimental exercise may be far from its relevance for immediate practice. Further work should be followed both theoretically and experimentally, to facilitate more discussions. As discussed, for instance, one question relates to the optimal size of the population to which the mechanism should be applied. There must be a trade-off relationship between the costs of providing common knowledge and the benefits of the mechanism applied to a larger population. Future work is needed to examine whether citizens would actually vote for the adoption of the penalty lottery compared to conventional fine mechanisms when society demands deterrence.

References

Ai, Chunrong and Edward C. Norton, "Interaction terms in logit and probit models," *Economics Letters*, 2003, *80* (1), 123–129.

Ambrus, Attila and Ben Greiner, "Imperfect Public Monitoring with Costly Punishment: An Experimental Study," *American Economic Review*, December 2012, *102* (7), 3317–3332.

Becker, Gary S., "Crime and Punishment: An Economic Approach," *Journal of Political Economy*, 1968, *76* (2), 169–217.

Bhattacharya, Sourav, John Duffy, and Sun-Tak Kim, "Compulsory versus voluntary voting: An experimental study," *Games and Economic Behavior*, 2014, *84* (Supplement C), 111–131.

Cramton, Peter, R. Richard Geddes, and Axel Ockenfelds, "Set road charges in real time to ease traffic," *Nature*, 2018, *560*, 23–25.

_ , _ , and _ , "Using Technology to Eliminate Traffic Congestion," *Journal of Institutional and Theoretical Economics*, 2019, *175* (1), 126–139.

Crosetto, Paolo and Antonio Filippin, "The "Bomb" Risk Elicitation Task," *Journal of Risk and Uncertainty*, Aug 2013, *47* (1), 31–65.

Drago, Francesco, Roberto Galbiati, and Pietro Vertova, "The Deterrent Effects of Prison: Evidence from a Natural Experiment," *Journal of Political Economy*, 2009, *117* (2), 257–280.

Duffy, John and Alexander Matros, "On the Use of Fines and Lottery Prizes to Increase Voter Turnout," *Economics Bulletin*, 2014, *34* (2), 966–975.

Feller, William, *An Introduction to Probability Theory and its Applications*, 3rd ed., Vol. 1, Wiley, 2008.

Filiz-Ozbay, Emel, Jonathan Guryan, Kyle Hyndman, Melissa S. Kearney, and Erkut Y. Ozbay, "Do lottery payments induce savings behavior? Evidence from the lab," *Journal of Public Economics*, 2015, *126*, 1–24.

Garrett, Thomas A. and Gary A. Wagner, "Red Ink in the Rearview Mirror: Local Fiscal Conditions and the Issuance of Traffic Tickets," *The Journal of Law and Economics*, 2009, *52* (1), 71–90.

Gerardi, Dino, Margaret A. McConnell, Julian Romero, and Leeat Yariv, "Get Out the (Costly) Vote: Institutional Design for Greater Participation," *Economic Inquiry*, 10 2016, *54* (4), 1963–1979.

Hillsman, Sally T., "Fines and Day Fines," *Crime and Justice*, 1990, *12*, 49–98.

Hjalmarsson, Randi, "Crime and Expected Punishment: Changes in Perceptions at the Age of Criminal Majority," *American Law and Economics Review*, 12 2008, *11* (1), 209–248.

Huck, Steffen and Georg Weizsäcker, "Risk, Complexity, and Deviations from Expected-Value Maximization: Results of a Lottery Choice Experiment," *Journal of Economic Psychology*, 1999, *20* (6), 699–715.

Imas, Alex, "The Realization Effect: Risk-Taking after Realized versus Paper Losses," *American Economic Review*, August 2016, *106* (8), 2086–2109.

Kearney, Melissa S., Peter Tufano, Jonathan Guryan, and Erik Hurst, "Making Savers Winners: An Overview of Prize-Linked Savings Products," Working Paper 16433, National Bureau of Economic Research October 2010.

Kim, Duk Gyoo, "Vaccination Lottery," *Economics Letters*, 2021, *208*, 110059.

Kimbrough, Erik O. and Alexander Vostroknutov, "Norms Make Prefernces Social," *Journal of the European Economic Association*, 2016, *14* (3), 608–638.

Makowsky, Michael D. and Thomas Stratmann, "Political Economy at Any Speed: What Determines Traffic Citations?," *The American Economic Review*, 2009, *99* (1), 509–527.

_ **and** _ , "More Tickets, Fewer Accidents: How Cash-Strapped Towns Make for Safer Roads," *The Journal of Law and Economics*, 2011, *54* (4), 863–888.

Martínez-Marquina, Alejandro, Muriel Niederle, and Emanuel Vespa, "Failures in Contingent Reasoning: The Role of Uncertainty," *American Economic Review*, October 2019, *109* (10), 3437–3474.

Morgan, John, "Financing Public Goods by Means of Lotteries," *The Review of Economic Studies*, 2000, *67* (4), 761–784.

— **and Martin Sefton**, "Funding Public Goods with Lotteries: Experimental Evidence," *The Review of Economic Studies*, 2000, *67* (4), 785–810.

Shoup, Donald, *The High Cost of Free Parking: Updated Edition*, Routledge, 2017.

Stigler, George J., "The Optimum Enforcement of Laws," *Journal of Political Economy*, 1970, *78* (3), 526–536.

Tversky, Amos and Daniel Kahneman, "The Framing of Decisions and the Psychology of Choice," *Science*, 1981, *211* (4481), 453–458.

Zizzo, Daniel J., "Experimenter Demand Effects in Economic Experiments," *Experimental Economics*, Mar 2010, *13* (1), 75–98.

A Appendix: Proofs

Approximation of k Consecutive Accumulations

First I introduce the method for calculating the probability that a success run of at least length k for an event with a probability of q occurs at least once within n trials. Regarding a success run of length $\kappa \in \{0,1,\ldots,k\}$ as a state, we can construct the following transition probability matrix, T of order $k+1$.

$$T = \begin{bmatrix} 1-q & q & 0 & \cdots & 0 \\ 1-q & 0 & q & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1-q & 0 & 0 & \cdots & q \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix},$$

where the probability of moving from state i to state j in one time period is $Pr(j|i) = T_{i+1,j+1}$. For example, $T_{23} = q$ is the probability of moving from a success run of length 1 to a success run of length 2. Then the entities of T^n are the probabilities of transitioning from one given state to another state in n Bernoulli trials. We call a Markov chain absorbing if there is at least one state such that the chain can never leave that state once entered. That is, k is an absorbing state. A stationary probability vector π, which does not change under application of the transition matrix, is $(0,0,\ldots,1)$. That is, $\pi T = \pi$ and $\lim_{n\to\infty}(T^n)_{1,k+1} = 1$.

Although this is the exact method to find the probability, Feller (2008) provides the approximation of the probability that a success run of at least length k for an event with a probability of q occurs at least once within n trials:

$$q_n \sim 1 - \frac{1-qx}{(k+1-kx)(1-q)} \frac{1}{x^{n+1}},$$

where q_n is the probability of a success run of at least length k in n trials, and x is the root of

$$Q(x) = 1 - x + (1-q)q^k x^{k+1} = 0,$$

other than $x = \frac{1}{q}$. The other root is smaller than $\frac{1}{q}$ if $(1-q)(k+1) > 1$. To check this, we need to show

$$x^* = \arg\min_x Q(x) < \frac{1}{q}$$

because for any $x > \frac{1}{q}$, $Q(x)$ is increasing. $x^* = \left(\frac{1}{(1-q)q^k(k+1)}\right)^{1/k}$, and this is strictly smaller than $\frac{1}{q}$ if and only if $\frac{1}{(1-q)(k+1)} < 1$. When $(1-q)(k+1) > 1$, the other root must be greater than 1 because $Q(1) = (1-q)q^k$, but it is typically close to 1. $Q(1) = (1-q)q^k$ tends to be small for a large k. This approximation renders an easier way to illustrate the theoretical predictions when citizens are homogeneous.

Proof of Proposition 1:

We apply the approximation of k consecutive accumulations to the relevant situations that we have in mind: a sequential public bad production in a society of risk-neutral homogeneous citizens. Suppose n violators (among n/p wrongdoers) sequentially arrive, and each of the violators passes over his/her fine to the next violator with probability q and pays all the accumulated fines with probability $1-q$. The probability that the fines are accumulated at least for k consecutive times is approximately $1 - \frac{1-qx}{(k+1-kx)(1-q)} \frac{1}{x^{n+1}}$. Since every citizen violates the law whenever it is beneficial to do so, the probability that a wrongdoer exists after n/p periods is

$$P(V|t = n/p) = \frac{1-qx}{(k+1-kx)(1-q)} \frac{1}{x^{n+1}}.$$

For the cases where we consider, the condition $(1-q)(k+1) > 1$ always holds because

$$k = \left\lceil \frac{B+C}{Fp(1-q)} \right\rceil - 1 \geq \frac{B+C}{Fp(1-q)} - 1 \Leftrightarrow (1-q)(k+1) \geq \frac{B+C}{Fp} > 1.$$

Note $B + C > Fp$, or $(1-p)B + p(B-F) > -C$ must hold, otherwise the assumption $(1-p)u_i(B) + pu_i(B-F) > u_i(-C)$ is violated. Since x is greater than 1, $P(V|n/p)$ decreases as n gets larger. However, if $q = 0$, $Q(x) = 1-x$, so $x = 1$. If $x = 1$, $P(V|t)$ is always 1 for any t.

Now we consider heterogeneous citizens. k_i is the smallest integer such that

$$(1-p+pq)u_i(B_i)-p(1-q)u_i(k_iF+F) \leq u_i(-C).$$

Since $u_i(\cdot)$ is weakly concave, the upper bound of such k_i is $\left\lceil \frac{B_i+C_i}{p(1-q)F} \right\rceil -1$. Thus $\bar{k} = \max_i k_i$ is finite for any $q \in [0,1)$. Let $G(k)$ denote the cumulative density function of $k = 0,\ldots,\bar{k}$, and $g(k) = G(k)-G(k-1), k = 1,\ldots,\bar{k}$ is the probability mass of the citizens with $k_i = k$. By the assumption of the initial incentive of public bad production, $g(0) = G(0) = 0$.

The transition probability matrix with the density function G, T_G is a square matrix of order $\bar{k}+1$. For notational simplicity, denote $l = \bar{k}+1$. T_G is characterized as follows:

- $(T_G)_{1,1} = 1-pq$.
- $(T_G)_{i,1} = (1-G(i-1))p(1-q)$ for $i = 2,\ldots,l$.
- $(T_G)_{i,i} = G(i-1)+(1-G(i-1))(1-p)$ for $i = 2,\ldots,l$.
- $(T_G)_{i,i+1} = (1-G(i-1))pq$ for $i = 1,\ldots,l-1$.
- All the other entities are zero.

Note that $G(l) = 1$ and l is an absorbing state, so the last row of T_G is a standard unit vector whose last entity is 1. Since $T^t_{1,l}$ is the probability of transitioning from state 0 to state $\bar{k}$ in t trials, $P(V|t) = 1-T^t_{1,l}$. The stationary probability vector π is $(0,0,\ldots,1)$. That is, $\lim_{n\to\infty}(T^t_G)_{1,l} = 1$, or $\lim_{t\to\infty} P(V|t) = 0$. Next we want to show the monotonicity: $(T^{t+1}_G)_{1,l} \geq (T^t_G)_{1,l}$ for any t. Since $(T^{t+1}_G)_{1,l} = (T^t_G)_{1,l-1}(T_G)_{l-1,l} + (T^t_G)_{1,l}(T_G)_{l,l} = (T^t_G)_{1,l-1}g(\bar{k})pq + (T^t_G)_{1,l}$, $(T^{t+1}_G)_{1,l} \geq (T^t_G)_{1,l}$ if and only if $(T^t_G)_{1,l-1}g(\bar{k})pq \geq 0$.

Last, we show that the expected time to reach the absorbing state is finite. Let

$$T_G = \begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{0} & 1 \end{bmatrix},$$

where $\mathbf{Q}$ is a $\bar{k}$-by-$\bar{k}$ matrix, $\mathbf{R} = [0,\ldots,0,g(\bar{k})pq]'$ is a $\bar{k}$-by-1 vector, and $\mathbf{0}$ is an 1-by-$\bar{k}$ zero vector. Thus, $\mathbf{Q}$ describes the probability of transitioning from some transient state to another, and $\mathbf{R}$ describes the probability of transitioning from some transient state to the absorbing state. Since state $\bar{k}-1$ is the only state that has a positive probability of transitioning to the absorbing state, all the other entities in $\mathbf{R}$ except for the last one are zeros.

The expected number of visits to a transient state j starting from a transient state i

before being absorbed is described by the so-called fundamental matrix, denoted by $\mathbf{N}$.

$$\mathbf{N} = \sum_{k=0}^{\infty} \mathbf{Q}^k = (\mathbf{I}_{\bar{k}} - \mathbf{Q})^{-1},$$

where $(\mathbf{Q}^k)_{i,j}$ is the probability of transitioning from state i to j in exactly k periods, and $\mathbf{I}_{\bar{k}}$ is the $\bar{k}$-by-$\bar{k}$ identity matrix. Summing this for all k yields the expected number of visits to transient states. Since the determinant of $(\mathbf{I}_{\bar{k}} - \mathbf{Q})$ is nonzero, $\mathbf{N}$ is well-defined. The expected number of periods being absorbed when starting in state 0 is the first entry of the vector $\mathbf{t} = \mathbf{NJ}$, where $\mathbf{J}$ is a $\bar{k}$-by-1 vector of which entities are all 1, that is, $\sum_{i=1}^{\bar{k}} \mathbf{N}_{1,i}$, which is finite. □

Proof of Proposition 2:

Our goal is to characterize the first row of $\mathbf{N}$ in terms of $G(k)$, p and q. Since the sum of the first row of $\mathbf{N}$ yields the expected number of time periods to reach the absorbing state, and we showed the monotonicity in Proposition 1, characterizing $\mathbf{N}$ is the key to describe $P(V|t)$. One property of the absorbing Markov chain is the probability of being absorbed in the absorbing state is captured by

$$\mathbf{B} = \mathbf{NR},$$

where $\mathbf{B}_i$ is the probability of being absorbed when starting from transient state i. Since there is only one absorbing state, $\mathbf{B}$'s entries are all 1. Also, since $\mathbf{R} = [0, \ldots, 0, g(\bar{k})pq]'$, the last column of $\mathbf{N}$ is $[\frac{1}{(1-G(\bar{k}-1))pq}, \ldots, \frac{1}{(1-G(\bar{k}-1))pq}]'$. Another important feature of the transition probability matrix considered in this paper is that state $i-1$ is the only transient state to reach state i in the next period, $i = 1, \ldots, \bar{k}-1$. Therefore we can recursively calculate $\mathbf{N}_{1,i}$ from $\mathbf{N}_{1,\bar{k}}$. For example, the expected number of visits to state $\bar{k}-2$, $\mathbf{N}_{1,\bar{k}-1}$ must be equal to $\mathbf{N}_{1,\bar{k}}(1-G(\bar{k}-1))\frac{1}{(1-G(\bar{k}-2))q}$, which is $\frac{1}{1-G(\bar{k}-2)pq^2}$. Thus, we have

$$\mathbf{N}_{1,i} = \frac{1}{(1-G(i-1))pq^{\bar{k}-(i-1)}}, \quad \text{for } i = 1, \ldots, \bar{k}.$$

$N_{1,1} = \frac{1}{pq^{\bar{k}}}$ is intuitive because $\frac{1}{q}$ is the inverse of the probability of transitioning from the other $\bar{k}-1$ transient state to another state than state 0, and $\frac{1}{pq}$ is the inverse of the probability of leaving state 0. Therefore the expected number of time periods being absorbed when starting in state 0 is

$$\sum_{k=1}^{\bar{k}} \mathbf{N}_{1,k} = \frac{1}{pq} \sum_{i=0}^{\bar{k}-1} \frac{1}{(1-G(i))q^i}.$$

Now we consider $\tilde{G}(k)$, a mean-preserving spread of $G(k)$ with a larger support. Without loss of generality, we can construct $\tilde{G}(k)$ by

$$\left\{ (\tilde{g}(1),\ldots,\tilde{g}(\bar{k}),\tilde{g}(\bar{k}+1)) \in \Delta \,\middle|\, \tilde{g}(i) = g(i) - \varepsilon_i \text{ for } i \le \bar{k}, \tilde{g}(\bar{k}+1) = \frac{1}{\bar{k}+1}\sum_{i=1}^{\bar{k}} i\varepsilon_i \right\},$$

where $\varepsilon_i \ge 0$. Then $1-\tilde{G}(i)$ is slightly larger than $1-G(i)$ for $i = 1,\ldots,\bar{k}$, so $\sum_{i=0}^{\bar{k}-1} \frac{1}{(1-\tilde{G}(i))q^i} < \sum_{i=0}^{\bar{k}-1} \frac{1}{(1-G(i))q^i}$, but $\frac{1}{g(\bar{k}+1)q^{\bar{k}+1}}$ is substantially larger than $\sum_{i=0}^{\bar{k}-1}\left[\frac{1}{(1-G(i))q^i} - \frac{1}{(1-\tilde{G}(i))q^i}\right]$ due to Jensen's inequality. □

B Appendix: Sample Instructions

[*Sample instructions for the Mq Treatment.]

This is an experiment in group decision making. Please pay attention to the instructions. You may earn a considerable amount of money which will be paid in cash at the end of the experiment. The currency in this experiment is called 'tokens'. In the beginning, you are endowed with 100 tokens.

There will be a quiz after the instructions, to ensure you understand the experiment.

Overview:

The experiment consists of 60 group decision-making 'rounds'. In each round, one of the group members decides (while others wait) to choose either a red ball or a blue ball, with knowing how many black stickers are in the common pool. The details follow.

How the groups are formed:

All subjects will be randomly assigned to groups of four. For example, if there are 20 subjects in this lab, there will be five groups of four subjects. You will belong to the same group throughout the whole experiment. There will be neither physical reallocation nor interactions. Only the server computer knows who are grouped with whom, and you input your choices to your computer interface. That is, you will not know who your group members are, and your group members will not know you either.

The balls:

In each round, one of the group members will choose either a red ball or a blue ball, while other members will see 'Please Wait' screen. What happens in your turn are as follows:

When you choose a red ball, nothing will happen further. You keep it and your group moves on to the next round.

When you choose a blue ball, nothing will happen with probability 0.7. With the other probability 0.3, a black sticker will be attached to the blue ball. If you have a black sticker, the computer will determine where black stickers go. With probability 0.5, the black sticker is removed from your blue ball, and added to the common pool shared by group members. With the other probability 0.5, you will keep your black sticker as well as all the black stickers from the common pool. That is, if you choose a blue ball, the probability that you will have black sticker(s) is 0.15 (= 0.3*0.5). Then your group moves on to the next round.

The balls and the sticker have different values. 1 token per each red ball will be deducted from your account, 5 tokens per each blue ball will be added to your account, and 8 tokens per each black sticker will be deducted from your account at the end of the session.

[Example] Suppose there are no black stickers in the common pool. If you choose a red ball in your turn, you lose 1 token. If you choose a blue ball and it does not have a black sticker, you earn 5 tokens. If the blue ball has a black sticker, then with probability 0.5 you earn 5 tokens and the sticker is added to the common pool. (Now 1 sticker is in the common pool.) With the other probability 0.5, you lose 3 tokens (earn 5 tokens from the blue ball, but lose 8 tokens from the black sticker).

Information in your turn:

When it is your turn, you will receive the following information:

- The number of black stickers in the common pool
- The numbers of red balls, blue balls, and black stickers you have kept

It will not be informed what other participants have done. If there are no black stickers in the common pool, it could mean either that no one has added black stickers to the common pool, or that someone ahead of you kept all the stickers from the common pool.

Final Earnings:

You are endowed with 100 tokens. If you have x red balls, y blue balls, and z black stickers at the end of the session, your total payoff is $100 - x + 5y - 8z$. Your earning will be converted into Euros at the rate of 8 eurocents per token. At the end of the main session, there might be an additional task of which result would be paid. The server computer will calculate the final payment. Please don't worry about this conversion.

Summary of the process:

1. The experiment will consist of 60 rounds. Everyone is endowed with 100 tokens.

2. In the beginning, all subjects will be randomly assigned to groups of four members. One of the group members will make a decision in each round.

3. When it is your turn, you will choose either a red ball or a blue ball, with knowing how many black stickers are in the common pool.

4. If you choose a red ball, your turn ends. If you choose a blue ball, with probability 0.3 a black sticker is followed. When a black sticker is followed by the blue ball, the black sticker will be detached from your blue ball and added to the common pool with probability 0.5. With the other probability 0.5, you will keep all the black stickers including those in the common pool.

5. A red ball is worth −1 token. A blue ball is worth +5 tokens. A black sticker is worth −8 tokens.

Quiz

Q1. Suppose 1 black sticker is in the common pool in your turn. You choose a red ball. Which of the followings is correct? (a) With some probability, it will come with a black sticker. (b) I keep the red ball. It will add 1 token to my account. (c) I keep the red ball. It will deduct 1 token from my account. (d) I keep the red ball and the black sticker in the common pool.

Q2. Suppose 2 black stickers are in the common pool in your turn. You choose a blue ball, and it does not come with a black sticker. Which of the followings is correct? (a) I keep the blue ball and one black sticker from the common pool. (b) I can decide how many black stickers I can add to the common pool. (c) I keep the blue ball. It will add 5 tokens to my account. (d) I keep the blue ball. It will deduct 8 tokens from my account.

Q3. Suppose 2 black stickers are in the common pool in your turn. You choose a blue ball, and it comes with a black sticker. Which of the followings is NOT correct? (a) With probability 0.5, I can change my choice. (b) With probability 0.5, I keep the blue ball, and the black sticker goes to the common pool. (c) With probability 0.5, I keep the blue ball and all the black stickers. (d) If the black sticker goes to the common pool, there are 3 stickers in the common pool.

Q4. If you choose a blue ball, what's the probability of keeping black sticker(s) in your account?

Q5. Suppose that you keep 10 blue balls and 2 black stickers at the end of the session. During the session, how many tokens have you gained/lost in total? (Don't count the endowment.)