

Collective Proofreading and the Optimal Voting Rule

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Abstract

Policy decisions often involve a repeated proofreading process before implementation. We present a dynamic model of proofreading decisions by a heterogeneous committee, in which the committee decides when to stop proofreading and implement a risky policy. The proofreading process is costly but necessary because the risky policy contains an unknown number of errors, and the value of the policy decreases by the number of undetected errors. Proofreading continues as long as a qualified majority votes for continuation. Once the proofreading process ends and the policy is implemented, members receive heterogeneous penalties based on the remaining errors. We characterize the optimal voting rule given the costs and penalties for the committee. We find that any qualified voting rule for proofreading results in an inefficient outcome. Unlike the result in [Strulovici \(2010\)](#), majority rule could have a bias not only toward under-experimentation but also toward over-experimentation.

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1 Introduction

This paper concerns an assessment process before implementing a new policy or a project. Before implementing a new policy, agencies frequently run a final evaluation

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process to verify its effectiveness. This process can be modeled as a proofreading process (e.g., [Yang et al., 1982](#); [Ferguson and Hardwick, 1989](#)), where a primary goal of the process is to find and fix as many errors as possible to make sure that the policy is successful. For example, suppose that an environmental protection agency is concerned about a new manufacturing process that may be harmful to the environment. The agency may conduct a series of investigations to detect violations of environmental laws such as overuse of toxic chemicals and pollutants in the process. The agency approves the manufacturing process only if the agency expects that the costs of further investigations outweigh the benefits of detecting potentially unobserved violations.

More complicated yet, the proofreading decisions are often made by a committee. When a decision committee consists of heterogeneous members, some members may want to conduct a stringent proofreading process, although it would incur a high cost of an investigation. On the contrary, others may be less involved with the investigation and may want fast approval of a new policy to enjoy potential benefits from policy implementation. To resolve this conflict of interest, the committee members might impose a collective decision rule, which is supposed to be designed to maximize social welfare. To understand the incentive structure of such a collective proofreading process and find an optimal voting rule, we build a model of collective proofreading decisions by heterogeneous members.

In our model, there is a committee consisting of n members, and it sequentially decides whether to continue a costly proofreading process or to stop it and implement the policy. The policy is risky in that it may contain errors, and its value decreases by the number of undetected errors. The committee continues rectifying the errors through the proofreading process as long as a qualified majority of members agrees to do so.¹

We first consider a situation in which the errors are realized from a Poisson distribution so that the proofreading level preferred by each member can be constant regardless

¹It would be more realistic to consider two-stage voting procedures: one vote for the proofreading stringency, and the other one for approval of a (potentially) risky policy over a status quo policy. We mainly focus on the inefficiency involved in the proofreading stage, but we admit that additional voting stage with a status quo policy can be another cause of potential inefficiency.

of the number of previously detected errors. To derive a clear policy implication, we assume further that there is at least one member whose optimal proofreading level as a single decision maker coincides with the socially optimal level. Hence, it is optimal for the committee to delegate the decision to this *representative* member. However, the representative member's optimal number of proofreading steps can be different from the number of proofreading steps approved by the qualified number of committee members. Due to this discrepancy, any qualified majority rule may result in a socially sub-optimal outcome.

The optimal voting rule that maximizes social welfare varies by the nature of the heterogeneity of each committee member. By letting e be the number of members who prefer to stop the proofreading process no earlier than the representative member, we find that the e -qualified majority rule is welfare-maximizing. If such e is less than half of the full committee, then it means the proofreading procedure should continue even when a minority of members want to continue under the optimal voting rule. In other words, the simple majority rule may have a bias toward under-experimentation, which follows the main finding of [Strulovici \(2010\)](#). However, we find a condition under which the simple majority rule results in an *over-experimentation* outcome, so the proofreading experimentation has to be carried out more conservatively. Consequently, a supermajority rule is required to achieve optimality. Therefore, we conclude that the simple majority rule could be socially optimal in the proofreading experimentation setting, but such optimality conditions are too restrictive to hold in reality.

We extend our model to address a situation in which there is a fixed number of issues that need to be evaluated. In particular, we assume that the errors are drawn from a Binomial distribution. In this case, the probability of detecting additional errors depends on the number of previously detected errors, so the optimal stopping decisions are no longer stationary. We can still characterize each committee member's optimal stopping rule as a function of the detected errors with respect to the proofreading levels. The same results as the Poisson model are retained.

1.1 Related Literature

Our model builds on seminal work on the single decision maker’s optimal proofreading problem by [Yang et al. \(1982\)](#). Previous studies on optimal proofreading decisions ([Chow and Schechner, 1985](#); [Ferguson and Hardwick, 1989](#); [Dalal and Mallows, 1988](#)) have focused on a single decision maker’s problem in which there are no concerns about a conflict of interest between heterogeneous committee members. In this paper, which benefits from a simple dynamic nature under the assumption of Poisson prior distribution of errors, we extend to a collective proofreading model without other behavioral and strategic concerns in collective decisions such as present-bias ([Jackson and Yariv, 2015](#)) or free-riding problems ([Keller and Rady, 2010](#)). As a result, we obtain a precise characterization of the welfare-maximizing voting rule.

This paper is also related to the recent literature on collective decisions in dynamic settings (e.g., [Strulovici, 2010](#); [Chan et al., 2018](#); [Jackson and Yariv, 2015](#); [Keller and Rady, 2010](#); [Lizzeri and Yariv, 2017](#); [Das et al., 2020](#)). In a modeling perspective, we consider agents who do not discount the future. This assumption is necessary for the existence of a representative agent in a dynamic model, as shown by [Jackson and Yariv \(2015\)](#). In our model, having a representative agent is not crucial, but it renders a more precise illustration of the welfare loss of a qualified majority rule compared with the socially optimal voting rule. In a setting of collective deliberation, [Lizzeri and Yariv \(2017\)](#) compare the performance of different voting rules in terms of committee welfare under the presence of a self-control problem, while we find an optimal voting rule without a self-control problem. [Das et al. \(2020\)](#) examine a situation where two players choose effort levels with two-armed exponential bandits and show that all Markov perfect equilibria imply the same amount of experimentation. Unlike theirs, the model considered in our paper takes the heterogeneous cost parameters as given so that the effort choice is binary.

[Strulovici \(2010\)](#) considers situations where individuals learn their type more accurately by experimentation, but our model deals with experimentation decisions under

complete information about the committee's heterogeneity. While the main finding of [Strulovici \(2010\)](#) is that majority rule has a downward bias in terms of experimentation, we found that majority rule could also yield a bias toward over-experimentation in our context. This observation is distinct from recent studies that attempt to link collective decisions to present-biased outcomes.

In the sense that the committees decide to stop searching for potential errors by proofreading, this paper is also related to studies on collective search by committees. In the context of accepting a proposal or searching for alternatives, [Compte and Jehiel \(2010\)](#) examine how each committee member affects the set of possible agreements. We could conduct the same exercise in our context, but our focus is more on the description of the socially optimal voting rule. [Albrecht et al. \(2010\)](#) compare how the collective search problem differs from a single-agent search problem with a focus on the case of symmetric agents, whereas our paper explicitly considers heterogeneous agents.

In [Moldovanu and Shi \(2013\)](#), a committee decides whether to accept the current alternative with multiple attributes or to continue the costly search, and each committee member can privately assess the quality of only one attribute. Our model can be considered as a costly search of alternatives (with one attribute) whose quality is non-decreasing over time. Although information aggregation and adverse selection problems under private information of committee members as in [Lauermann and Wolinsky \(2016\)](#) are worth being investigated, we focus on the case of complete information.

Our paper contributes to advancement of theoretical models on environmental policy choice in different economic settings. [Viscusi and Zeckhauser \(1976\)](#) provide a tractable framework to deal with a situation in which there is no clear ranking among environmental policies due to the presence of uncertainty. In particular, their model conveniently examines how a chance of irreversibility affects policy choice, as discussed in [Viscusi \(1985\)](#). [Wirl \(2006\)](#) examines how irreversibilities of environmental policy affect the optimal intertemporal accumulation of greenhouse gases in the atmosphere under uncertainty. [Gsottbauer and van den Bergh \(2011\)](#) investigate the relationship between

environmental policy decisions and other-regarding preferences.

In our paper, we consider a different nature of the policy implementation: the uncertainty of the benefit (damage) of the policy and conflicts of interest between heterogeneous committee members resolved by voting. Although applications in environmental policy decisions motivate this paper, the model is applicable in many other cases such as optimal R&D investments (Moscarini and Smith, 2001; Weeds, 2002) decided by a committee.

2 Benchmark Model

In this section, we describe a single decision maker's choice problem. We introduce key assumptions simplifying the problem. Specifically, we assume that the number of errors is drawn from a Poisson distribution. Then, we investigate an optimal proofreading strategy, which is a building block for analyzing equilibrium behavior in the committee decision.

2.1 Setup

There is a risky project containing \mathbf{M} errors, where \mathbf{M} is a random variable distributed over \mathbb{N}_0 with $\mathbb{E}[\mathbf{M}] < \infty$.² There is a single decision-maker who tries to find and correct errors through a *proofreading process*. In each period t , the decision maker decides whether to stop or continue the proofreading process. If he decides to stop the process, he pays a penalty of $D > 0$ for each remaining error. Each error-finding step incurs a cost $c > 0$. To ignore the time preference of the decision maker, we assume that he does not discount the future, or that the time gap between each proofreading step is minuscule.

We let \mathbf{X}_t be the random variable representing the number of detected errors in period t . We denote by X_t the realized number of detected errors in period t , $X_t \geq 0$, and

²Throughout the paper, \mathbb{N}_0 represents the set of natural numbers including zero.

$\sum_{j=1}^t X_j \leq \mathbf{M}$ for all $t \geq 1$. We assume the following for the number of errors \mathbf{M} and the sequence of detected errors $(X_t)_{t \geq 1}$:³

Assumption 1. For \mathbf{M} and $(X_t)_{t \geq 1}$,

(1) the number of errors follows a Poisson distribution with $\lambda > 0$:

$$\mathbf{M} \sim \text{Poisson}(\lambda).$$

(2) the number of detected errors in period $t + 1$ follows a Binomial distribution:

$$\mathbf{X}_{t+1} | \mathbf{M}, X_1, \dots, X_t \sim \text{Binomial} \left(\mathbf{M} - \sum_{j=1}^t X_j, p \right),$$

where $p \in (0, 1)$ is the probability of detecting an error.

As shown in Lemma 3.1 in [Ferguson and Hardwick \(1989\)](#), Assumption 1 dramatically simplifies the dynamic nature of the problem as the conditional probability of having errors in period $t + 1$ is independent of the history of previous error findings,⁴ which is recapitulated by the following lemma.

Lemma 1. The number of remaining errors after t steps of proofreading follows a Poisson distribution with $\lambda(1 - p)^t$:

$$\mathbf{M} - \sum_{j=1}^t X_j \sim \text{Poisson}(\lambda(1 - p)^t).$$

[Lemma 1](#) implies (1) that the mathematical expectation of the number of remaining errors is $\lambda(1 - p)^t$, which is decreasing in t , and (2) that the expectation is independent of a history of error detection.

³We consistently use boldface mathtype letters (e.g., \mathbf{M} and \mathbf{X}_{t+1}) to indicate random variables and normal mathtype letters (e.g., X_j) as a realization of a relevant random variable. Thus an expression, for example, $\mathbf{M} - \sum_{j=1}^t X_j$ is a short form of $\mathbf{M} - \sum \mathbf{X}_j | \mathbf{X}_1 = X_1, \mathbf{X}_2 = X_2, \dots, \mathbf{X}_t = X_t$.

⁴When the number of detected errors follows a Binomial distribution, the conditional probability does depend on the previous errors found. More details can be found in [Appendix B](#).

To make the decision maker's problem non-trivial, we assume that (at least one) proofreading is ex-ante desirable. Since the expected penalty from approval of the risky project without proofreading is calculated as $D\lambda$, we assume that proofreading once is better than doing nothing:

Assumption 2. *Proofreading is desirable:*

$$D\lambda > D\lambda(1-p) + c \Leftrightarrow p\lambda > \frac{c}{D}.$$

2.2 Optimal Strategy

We analyze the single decision maker's optimal proofreading strategy. At time t , she observes the number of detected errors by $t-1$, and decides whether to continue or stop the proofreading process.

After t steps of the proofreading process, the expected penalty from implementing the policy is:

$$D\mathbb{E}[\text{remaining errors}|\text{history}] = D\mathbb{E}\left[\mathbf{M} - \sum_{j=1}^t X_j\right] = D\lambda(1-p)^t,$$

where the second equality comes from [Lemma 1](#). The total expected cost, the sum of the expected penalty and proofreading costs of t steps, is calculated as

$$C(t) = D\lambda(1-p)^t + tc.$$

The decision maker's key incentives are illustrated in [Figure 1](#). The blue triangles denote the expected penalty of approving the risky project in period t after t error-finding trials. The expected penalty is strictly decreasing in t because the decision maker becomes more confident with the risky policy as a longer proofreading process is conducted.

The black dots represent the total expected cost of approving the risky project after t steps of proofreading. It initially decreases in t by [Assumption 2](#) because proofreading decreases the number of remaining errors and the probability of having errors, and this

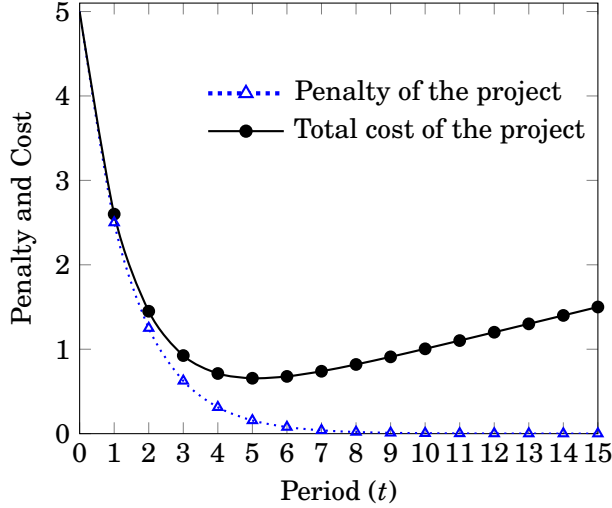


Figure 1: A numerical example of the costs: $D = 1$, $\lambda = 5$, $p = \frac{1}{2}$, and $c = 0.1$.

decrease dominates the constant marginal cost of additional proofreading. However, after a certain period ($t = 5$ in the figure), the total expected cost increases because the marginal benefit of having fewer errors is dominated by the marginal cost of additional proofreading. We denote t^* as the period minimizing the total expected cost by ignoring the integer value requirement of it.

Although it is obvious in the single decision-making problem, it will be useful in the context of the collective decisions to note that the decision maker has an incentive to continue the proofreading process during the periods in interval $[0, t^*]$ rather than stop and approve the risky project immediately because

$$D\lambda(1-p)^t \geq D\lambda(1-p)^{t^*} + (t^* - t)c.$$

The optimal stopping time is determined by the cost-penalty ratio, $\frac{c}{D}$. To see this point, note that the marginal decrease of the expected number of errors must be the same as the cost-penalty ratio at the optimal step t^* if it were to be on \mathbb{R}_+ :

$$\lambda(1-p)^{t^*} \ln\left(\frac{1}{1-p}\right) = \frac{c}{D}.$$

Note that the interior solution of the above equation exists by Assumption 2. As the cost-penalty ratio decreases, the optimal stopping time t^* increases. Thus, the cost-penalty ratio is a key characteristic that represents the decision maker's preference over stopping times.

3 Collective Proofreading

Our main contribution is to extend the previous single decision maker's problem to a collective deliberation problem. We first present a collective proofreading environment. Then, we introduce strategies and equilibrium notion under the environment.

Collective proofreading environment. Time is discretized as $t = 1, 2, 3, \dots$. There is a decision committee consisting of $n \geq 3$ members, where n is assumed to be odd. Each member is indexed by $i \in N = \{1, \dots, n\}$. At the outset of the proofreading process, the number of errors is drawn by nature from a Poisson distribution with parameter $\lambda > 0$. In the collective proofreading process, each error is detected with probability p if the committee proofreads the error.

As a collection of collective decision rules, we consider a qualified majority rule. Specifically, under a q -rule, the committee continues the proofreading process if at least q members of the committee want to do so.⁵ We assume that members' previous voting history is publicly observable across the members. We refer to a $\frac{n+1}{2}$ -rule as a simple majority rule.

The collective proofreading process begins with a common prior belief about the number of errors, given by the parameter λ with detection probability p . Thereafter, the committee members learn about the number of remaining errors in a Bayesian fashion by observing one another's voting actions and the detected errors. Since the voting history is publicly observable, the members share common posterior beliefs throughout the proof-

⁵We note here that we have not yet defined the set of strategies available to each member. We will soon explain strategy profiles and the induced proofreading process by imposing Markov perfection.

reading process. Figure 2 illustrates the intermediate collective proofreading process in period t .

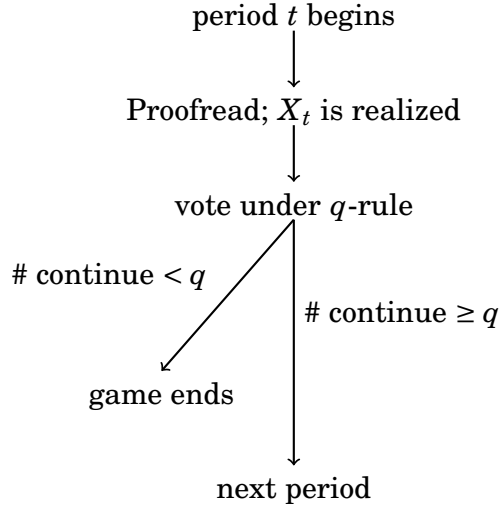


Figure 2: Illustration of the timing of the game in a period

Committee members are heterogeneous in their penalty per remaining error and the proofreading cost. This heterogeneity may be due to the members' different positions in the agency or political interests. For instance, politicians and policymakers may have small proofreading costs because they are less involved with the actual proofreading process compared to engineers and researchers who actually spend their time and effort to investigate the policy. The penalty per remaining error would also be heterogeneous: The investigator of the environmental policy would be less affected by the resulting misconduct of the new manufacturing process, while the (representative of) residents of the affected area may suffer critically from the resulting errors.

We denote by $\{(D_i, c_i)\}_{i=1}^n$ the heterogeneous committee members. Without loss of generality, we assume that $D_{j+1} \geq D_j > 0$ for $j = 1, \dots, n-1$. We let $\vec{t}^* = (t_i^*)_{i=1}^n$ be the vector of each member's optimal stopping time if they were to behave as a single decision maker in the same proofreading environment, and we denote by $m(\vec{t}^*)$ the median of the optimal stopping times. We assume a nontrivial amount of heterogeneity: $t_i^* \neq t_j^*$ for some $i \neq j$.

$C_i(t)$ represents member i 's total cost from t steps of proofreading, $D_i \lambda(1-p)^t + tc_i$.

The aggregate cost $C_{agg}(t)$ is defined as the sum of members' costs, and t_{agg}^* be the optimal stopping based on $C_{agg}(t)$. Then, a member $r \in N$ is said to be a **representative member** if t_r^* coincides with t_{agg}^* . We assume that there is at least one member whose cost function is aligned with the aggregate cost of the decision committee, which is considered to be the social cost function.⁶

Assumption 3. *There exists a representative member.*

Assumption 3 means that the aggregated cost is minimized if the committee delegates the decision to member r ,⁷ or construct a voting rule that makes the representative member pivotal. We call member r the representative member. All the intuitions and results do not rely on the index (identity) of the representative member.

It is worth noting that Assumption 3 is unnecessary to derive our main results. Even without this assumption, there must be someone whose preferred proofreading steps are closest to the socially optimal steps, and we can examine the inefficiency of collective proofreading with this near-representative member. Rather, Assumption 3 merely creates a situation where a definitive way to implement the social optimum exists.

Strategies and equilibrium. We assume that committee members use pure Markov strategies in discrete time with the posterior belief about the number of remaining errors as the state variable. Note that the number of remaining errors is invariant of detection history by Lemma 1. Thus, a pure Markov strategy is a sequence of voting behavior whether to vote to stop or continue in each period. We restrict our attention to the set of Markov strategies in which each member i chooses a finite threshold time $t_i \geq 2$ such that she votes to continue if and only if $t < t_i$.⁸ We call those strategies monotone Markov strategies.

⁶Here, we adopt a utilitarian criterion.

⁷To guarantee the existence of such a representative member, the assumption of no future discounting is crucial. Specifically, as shown in Jackson and Yariv (2020), it is impossible to have such a representative member if members are heterogeneous in their discount factors.

⁸The requirement of $t_i \geq 2$ is induced by Assumption 2. We also rule out the possibility of the infinite threshold; since a strategy profile consisting of the infinite threshold results in infinity cost of proofreading for everyone, the committee can easily reach a consensus that no member uses the infinite threshold.

It turns out that sincere voting is a weakly dominant strategy among the set of monotone Markov strategies. To see why, observe that voting for continuation until member i 's optimal stopping time t_i^* is at least as profitable as voting against continuation before t_i^* . Voting for continuation after t_i^* does not help attain the smallest expected cost either. Therefore, behaving truthfully as if a committee member were a single-decision maker is weakly dominant when voters use monotone strategies. As such, we analyze a Markov perfect equilibrium in which everyone behaves sincerely according to their stand-alone optimal stopping time. The following proposition formalizes what we have discussed so far:

Proposition 1. *There is a Markov perfect equilibrium consisting of weakly dominant strategies in which each member sincerely votes for continuation.*

Throughout the paper, we call this Markov perfect equilibrium the *equilibrium* unless otherwise stated. In the following section, we find an optimal voting rule assuming that voters act on the equilibrium.

4 Optimal Voting Rule

We now analyze the equilibrium outcomes under different q -rules. We identify the condition under which the q -rule is socially optimal.

Proposition 2. *In equilibrium, the q -rule produces the socially-optimal outcome if and only if q satisfies $q = \#\{i | t_i^* \geq t_r^*\}$.*

[Proposition 2](#) states that the socially-optimal voting rule should make the representative member pivotal. In other words, in any situations where the representative member is not pivotal, collective decisions yield socially inefficient outcomes.

The key intuition for the inefficiency is captured by the fact that member i 's optimal stopping time crucially depends on $\frac{c_i}{D_i}$, neither D_i nor c_i per se. Even if we impose

a monotone rank of c_i , along with $D_{i+1} \geq D_i$ for $i = 1, \dots, n-1$, the rank of $\frac{c_i}{D_i}$ is not sufficient to determine the optimal voting rule.

The following two examples illustrate situations in which the committee continues the proofreading steps too long and not long enough under a simple majority rule.

Member	D_i	c_i	t_i^*	$C_i(2)$	$C_i(3)$	$C_i(4)$	$C_i(5)$	$C_i(6)$	$C_i(7)$
1	1.0	0.60	3	2.45	2.43	2.71	3.16	3.68	4.24
2	1.1	0.30	4	1.98	1.59	1.54	1.67	1.89	2.14
3	1.2	0.15	5	1.80	1.20	0.98	0.94	0.99	1.10
4	1.7	0.10	6	2.33	1.36	0.93	0.77	0.73	0.77
5	2.0	0.05	7	2.60	1.40	0.83	0.56	0.46	0.43
Σ				11.15	7.98	6.99	7.09	7.75	8.67

Table 1: Inefficiency: Over-proofreading. $\lambda = 5, p = 0.5$

The example in Table 1 considers a situation where the costs are negatively aligned with the penalties (that is, $c_i \geq c_{i+1}$). This cost-penalty relationship could be observed in a committee in which senior members or authoritative members take responsibility for remaining errors while junior members exert effort to proofread. In this example, the order of $\frac{c_i}{D_i}$ is monotone, as is the optimal stopping time for each member. Although the preferences of each member's optimal stopping time are well ordered, and member 3 is seemingly a median member in every aspect, the simple majority rule involves over-proofreading.

Member	D_i	c_i	t_i^*	$C_i(2)$	$C_i(3)$	$C_i(4)$	$C_i(5)$	$C_i(6)$	$C_i(7)$
1	1.0	0.05	6	1.35	0.78	0.51	0.41	0.38	0.39
2	1.1	0.10	5	1.58	0.99	0.74	0.67	0.69	0.74
3	1.2	0.20	4	1.90	1.35	1.18	1.19	1.29	1.45
4	1.7	0.30	4	2.73	1.96	1.73	1.77	1.93	2.17
5	2.0	0.40	4	3.30	2.45	2.23	2.31	2.56	2.88
Σ				10.85	7.53	6.39	6.34	6.85	7.62

Table 2: Inefficiency: Under-proofreading. $\lambda = 5, p = 0.5$

The example in Table 2 considers a situation where the costs are positively aligned with the penalties (that is, $c_i \leq c_{i+1}$). This cost-penalty relationship can be observed in a

committee in which some members are more involved in the policy than other members. In this example, the simple majority rule involves under-proofreading.

Two observations are worth mentioning. First, the positive (resp. negative) relationship between D_i and c_i does not necessarily imply under-proofreading (resp. over-proofreading). The opposite cases are also possible. Second, the optimal voting rule may require a smaller number of votes than the simple majority to continue the proofreading process.

One remaining question would be under what conditions the simple majority rule is socially optimal.

Proposition 3. *A super-(sub-)majority rule is socially optimal if and only if*

$$\frac{c_m}{D_m} < (>) \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n D_i},$$

where the subscript m denotes the committee member whose $\frac{c}{D}$ is the median.

Proposition 3 also implies that the simple majority rule is socially optimal if and only if $\frac{c_m}{D_m} = \frac{\sum c_i}{\sum D_i}$.⁹ For example, if the cost parameters and damage parameters are symmetrically distributed, then c_m and D_m coincide with the average of c_i and D_i , respectively, and the simple majority rule is socially optimal. **Proposition 3** indirectly illustrates how fragile the foundation of the simple majority rule is in the context of collective proofreading decisions. It is well known that asymmetric intensities among voters may make the simple majority rule socially undesirable.¹⁰ It is even harder to achieve efficiency in the context of collective proofreading because we need symmetry in both dimensions.

Similar to the fact that a simple majority rule cannot be optimal for most cases, any q -rule cannot be a panacea for all cases. A naturally followed question is whether the committee could endogenously choose the optimal voting rule. In the ex-ante stage,

⁹More rigorously speaking, $\frac{c_m}{D_m} = \frac{\sum c_i}{\sum D_i}$ is a sufficient condition for the simple majority to be socially optimal, but it is not a necessary condition. Since q in the q -rule is a positive integer, the simple majority rule can still be socially optimal if $\left| \frac{c_m}{D_m} - \frac{\sum c_i}{\sum D_i} \right|$ is sufficiently small.

¹⁰For reviews of this line of research, see [Posner and Weyl \(2017\)](#).

where every member knows the joint distribution of the damages and costs but does not know their realized values, the committee unequivocally prefers the optimal voting rule, as it renders the highest expected payoff. Therefore, for any voting rules and protocols for mapping individuals' preference orders to the committee's preference order, the optimal voting rule will be selected. However, in the interim stage, where committee members privately learn their own values (D_i and c_i), their preferred stopping times, and hence their preferred voting rules, vary. In this case, the primitive voting rule to determine the voting rule (Barbera and Jackson, 2004) may make a deviation from the optimal voting rule. Thus, a normative suggestion we can draw from this section is that the voting rule should either be made at the ex-ante stage or be exogenously determined by an unbiased executor.

5 Concluding Remarks

Many serious policies and projects are implemented after rigorous proofreading steps. When a committee with heterogeneous members collectively makes decisions about the proofreading process, the simple majority rule is likely to result in an inferior outcome. The socially optimal voting rule, which requires the committee to continue the proofreading process up to the socially optimal proofreading level, can be constructed only after accounting for the members' heterogeneity. This construction is a nontrivial task: Although we are completely informed about each committee member's heterogeneity, there is no clear rule about which voting rule should be maintained. A simple majority rule could have a bias not only toward under-proofreading but also toward over-proofreading. The optimal voting rule should either be made at the ex-ante stage or be exogenously determined by an unbiased executor each time.

Two possible extensions are worth mentioning. First, one may consider a model in which some members prefer taking the safe (status quo) policy rather than choosing the risky policy after they agree to stop proofreading. That is, a committee follows two-stage

voting procedures: one vote for the proofreading stringency, and the other for approval of a risky policy over the status quo. In this case, the representative member or the decisive voter in the proofreading-decision stage may have to form a coalition with other members to implement the socially-optimal policy. For this analysis, one needs to model a dynamic bargaining process in line with the current proofreading process. Second, different types of errors in terms of the difficulty to detect could be considered. When obvious errors are easy to detect while subtle ones are difficult to catch, not only the probability of finding an error will decrease over time, but the expected penalties associated with the remaining errors would change accordingly.

Although the theory has many interesting predictions, still many empirical questions remain unanswered: Do subjects optimally vote to stop proofreading when the expected benefit exceeds the cost they can afford? Do they rationally respond to changes in the distribution of the errors of the risky project and the degree of committee heterogeneity? How much is the actual welfare loss when adhering the simple majority voting rules that are not maximizing welfare?¹¹ We may answer all the questions by conducting controlled laboratory experiments.

¹¹In a similar line of questioning, [Martinelli et al. \(2020\)](#) provide experimental evidence that a simple majority rule, which is supposed to be sub-optimal, performs better than the (theoretically) optimal voting rule.

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A Proofs

Proof of Lemma 1

Proof. Elementary computations show that $\mathbf{M} \sim \text{Poisson}(\lambda)$ and $\mathbf{X}_1 | \mathbf{M} \sim \text{Binomial}(\mathbf{M}, p)$ imply that (1) $\mathbf{X}_1 \sim \text{Poisson}(\lambda p)$, and (2) \mathbf{X}_1 and $\mathbf{M} - \mathbf{X}_1$ are independent. Thus, we have that $\mathbf{M} - X_1 \sim \text{Poisson}(\lambda(1 - p))$. Then, by the induction principle, it follows that $\mathbf{M} - \sum_{j=1}^t X_j \sim \text{Poisson}(\lambda(1 - p)^t)$ as stated in the lemma. \square

Proof of Proposition 1

Proof. We first show that the strategy of voting for continuation if and only if $t \leq t_i^*$ returns a payoff at least as high as any other strategies. Let t_{-i} be a fixed strategy profile of the members except member i . There are two cases to consider: (1) the proofreading never stops regardless of member i 's decision, and (2) the proofreading may stop at some period t depending on member i 's decision in period t . Thus, it suffices to consider the second case. Note that the members are assumed to use monotone strategies. Let t_1 be the first time in which member i 's decision matters. If $t_i^* \leq t_1$, then the desired strategy returns the highest payoff as it terminates the proofreading process at time t_1 . If $t_1 < t_i^*$, then it is better for member i to continue the process until t_i^* . Hence, the desired strategy never returns a strictly smaller payoff than any other strategies.

We now show that there exists at least one strategy profile of other players such that the desired strategy returns a strictly higher payoff with respect to the given strategy profile. Let t_{-i} be the strategy of the members except member i in which there are $q - 1$ number of members who vote to stop whenever $t \geq t_i^*$, and there are $n - q$ number of members who always vote for continuation. Then, it follows that member i 's decision is pivotal and playing the desired voting strategy returns a strictly higher payoff than any other strategies. \square

Proof of Proposition 2

Proof. The "only if" part is trivial: If q does not satisfy $q = \#\{i | t_i^* \geq t_r^*\}$, then such q -rule does not produce the socially-optimal outcome. If the q -rule produces the socially optimal outcome. Let $q = \#\{i | t_i^* \geq t_r^*\}$. To show that the q -rule produces the socially-optimal outcome in the equilibrium, it suffices to show that the proofreading process stops at time t_r^* . By [Proposition 1](#), voter i votes for continuation whenever $t \leq t_i^*$. For any $t \leq t_r^*$, there exist at least q members who vote for continuation. Thus, the proofreading stops at exactly period t_r^* , and so the proposition is proven. \square

Proof of [Proposition 3](#)

Proof. Since $\sum_{i=1}^n C_i(t) = (\sum_{i=1}^n D_i)\lambda(1-p)^t + t(\sum_{i=1}^n c_i)$, t_r^* , the minimizing argument of $\sum C_i(t)$, satisfies the following:

$$\lambda(1-p)^{t_r^*} \ln\left(\frac{1}{1-p}\right) = \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n D_i}.$$

It is straightforward to check if $\frac{c_m}{D_m} = \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n D_i}$, then t_m^* coincides with t_r^* because

$$\lambda(1-p)^{t_m^*} \ln\left(\frac{1}{1-p}\right) = \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n D_i} = \frac{c_m}{D_m} = \lambda(1-p)^{t_r^*} \ln\left(\frac{1}{1-p}\right).$$

If $\frac{c_m}{D_m} < \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n D_i}$, then $t_m^* > t_r^*$. That is, the median voter prefers to proofread more than the socially optimal proofreading level, and a simple majority has a bias toward over-experimentation. The case with $\frac{c_m}{D_m} > \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n D_i}$ is analogous. \square

B Binomial Model

In [Sections 2 and 3](#), thanks to the memoryless property of Poisson distributions, each member's preferred proofreading level was constant regardless of the number of previously detected errors. Since the support of the Poisson distribution is unbounded, the model we considered in the previous sections may be different from more realistic situ-

ations where a committee collectively proofreads finite issues. In this case, the number of previously detected (and fixed) errors do affect further proofreading decisions. In this section, we assume that the number of errors is drawn from a Binomial distribution.¹²

There is a single risky project containing $\mathbf{M} \sim \text{Binomial}(N, \pi)$ errors, where N is the number of issues that affect the overall return of the project, and $\pi > 0$ is the error probability for each issue that contains either one error or none.

Let $\mathbf{Y}_t = \mathbb{E}[\mathbf{M}_t | X_1, \dots, X_t]D + ct$ denote the total expected cost after t steps of proofreading, where X_j is the number of errors detected in period $1 \leq j \leq t$, and $\mathbf{M}_t = \mathbf{M} - \sum_{j=1}^t X_j$ is the number of remaining errors.

We assume independence of the error-detecting probability:

Assumption 4. *In each proofreading step, each error is detected with probability $p > 0$, and the detection probability is independent of the detection history and detection of other errors:*

$$\mathbf{X}_{t+1} | \mathbf{M}, X_1, \dots, X_t \sim \text{Binomial}(\mathbf{M}_t, p).$$

Then, by Lemma 4.1 in [Ferguson and Hardwick \(1989\)](#), we obtain the following:

Lemma 2. *The number of remaining errors \mathbf{M}_t follows a binomial distribution:*

$$\mathbf{M}_t | X_1, \dots, X_t \sim \text{Binomial}(N - \sum_{j=1}^t X_j, \pi_t),$$

where the updated error probability π_t is calculated as

$$\pi_t = \frac{\pi(1-p)^t}{(1-\pi) + \pi(1-p)^t}.$$

There are several notable properties driven from [Lemma 2](#). First, the updated error probability is independent of the previously detected errors. Second, the error probability

¹²A model with this assumption nests a situation where a committee collectively decides to draw an additional signal to learn the binary state of the world, as it could be understood as an error (bad state) drawn from a Bernoulli distribution.

π_t is monotone *decreasing* in t .¹³ Third, the expected number of remaining errors after t proofreading steps is history dependent:

$$\begin{aligned}
\mathbb{E}[\mathbf{M}_{t+1} | X_1, \dots, X_t] &= \mathbb{E}[\mathbf{M}_t - \mathbf{X}_{t+1} | X_1, \dots, X_t] \\
&= \mathbb{E}[\mathbf{M}_t | X_1, \dots, X_t] - \mathbb{E}[\mathbf{X}_{t+1} | X_1, \dots, X_t] \\
&= \left(N - \sum_{j=1}^t X_j \right) \pi_t - \mathbb{E}[\mathbb{E}[\mathbf{X}_{t+1} | \mathbf{M}, X_1, \dots, X_t] | X_1, \dots, X_t] \\
&= \left(N - \sum_{j=1}^t X_j \right) \pi_t - \mathbb{E}[\mathbf{M}_t p | X_1, \dots, X_t] \\
&= (1-p) \left(N - \sum_{j=1}^t X_j \right) \pi_t,
\end{aligned}$$

which depends on both (1) the updated error probability π_t and (2) the sum of detected errors up to period t . If other things are equal, the expected number of remaining errors is decreasing in both period t and the number of detected errors in previous periods. Consequently, the expected cost of stopping in period $t+1$ is

$$\begin{aligned}
\mathbb{E}[\mathbf{Y}_{t+1} | X_1, \dots, X_t] &= \mathbb{E}[\mathbf{M}_{t+1} | X_1, \dots, X_t] D + (t+1)c \\
&= (1-p) \left(N - \sum_{j=1}^t X_j \right) \pi_t D + (t+1)c.
\end{aligned}$$

The expected cost is now history dependent, as is the optimal strategy.

One assured condition is that the optimal strategy is still monotone. In fact, it is well-known in the optimal stopping literature that an optimal stopping strategy has a myopic form if a stopping problem is *monotone*.¹⁴ The monotonicity requires that the sets

$$A_t = \{\mathbf{Y}_t < \mathbb{E}[\mathbf{Y}_{t+1} | X_1, \dots, X_t]\}$$

¹³This probability directly follows from the fact that

$$\pi_t = \frac{\pi(1-p)^t}{(1-\pi) + \pi(1-p)^t} = \frac{\pi}{\frac{1-\pi}{(1-p)^t} + \pi}.$$

¹⁴It is also called the *one-stage look-ahead rule*. See, for example, [Bruss \(2000\)](#).

are monotone increasing as $A_t \subseteq A_{t+1}$ almost surely for any t . In other words, the condition $A_t \subseteq A_{t+1}$ means that if an immediate stop at time t is optimal in period t , then it is also optimal to stop at all the following future periods, no matter how the future errors are detected.¹⁵

In the current setting, the condition for monotonicity is satisfied. To see why, observe that

$$\begin{aligned} \mathbf{Y}_t < \mathbb{E}[\mathbf{Y}_{t+1} | X_1, \dots, X_t] &\iff \mathbb{E}[\mathbf{M}_{t+1} | X_1, \dots, X_t] p D < c \\ &\iff (1-p) \left(N - \sum_{j=1}^t X_j \right) \pi_t p D < c \\ &\iff (1-p) \left(N - \sum_{j=1}^t X_j \right) \frac{\pi}{\frac{1-\pi}{(1-p)^t} + \pi} p D < c. \end{aligned}$$

The second factor on the left-hand side of the last inequality, $N - \sum_{j=1}^t X_j$, is the difference between the total number of issues and the number of detected errors. This factor is non-increasing. The third factor, $\frac{\pi}{\frac{1-\pi}{(1-p)^t} + \pi}$, is the updated belief of having an error in each remaining issue. This factor is strictly decreasing and independent of the history of detected errors. Therefore, the whole expression on the left-hand side is decreasing.

The optimal stopping time t^* is the smallest integer t such that

$$\left(N - \sum_{j=1}^t X_j \right) \frac{(1-p)\pi p}{\frac{1-\pi}{(1-p)^t} + \pi} \leq \frac{c}{D},$$

and the cost-penalty ratio, c/D is still a key characteristic. From this, we can characterize the optimal stopping threshold as a pair of the number of detected errors and the decision period. After rearranging the inequality, we have

$$\sum_{j=1}^t X_j \geq \kappa_0 - \frac{\kappa_1}{(1-p)^{t+1}},$$

where $\kappa_0 = N - \frac{c}{Dp(1-p)}$ and $\kappa_1 = \frac{c(1-\pi)}{D\pi p}$. This stopping rule implies that proofreading

¹⁵Equivalently, when a member prefers stopping in period t to stopping in period $t+1$, then she also prefers stopping in period $t+1$ to stopping in period $t+2$.

is satisfied for all the points colored blue in the figure. Ultimately, due to this feature, the boundary between the blue dots and the black dots is decreasing in period t .

Each red line represents a sample path. Each path moves in a north-east direction in the lattice plane. The solid red line describes the scenario when $X_1 = 1, X_2 = 2, X_3 = 0, X_4 = 3$, and the decision maker stops at period 4. The dashed red line describes the scenario when $X_1 = 0, X_2 = 1, X_3 = 0, X_4 = 0, X_5 = 1$, and the decision maker stops at period 5.

If t and $\sum_{j=1}^t X_j$ were to be on \mathbb{R}_+ , then we can delineate the boundary to continue proofreading on the plane of $(t, \sum_{j=1}^t X_j)$. Unless otherwise stated, we refer to this boundary as the optimal stopping rule. Although the optimal stopping decision for proofreading becomes history dependent, the same results as the original model are retained.

Two key features of the optimal stopping rules are (1) that given any sample path (or history), the decision maker's optimal stopping rule is aligned with his/her cost-disadvantage ratio, and (2) that each heterogeneous committee member's rule does not cross another's rule. Committee member i with a high $\frac{c_i}{D_i}$ will always have a weakly lower boundary in terms of the sum of the detected errors. Since this stopping problem is monotone, it is natural to restrict our attention to the class of monotone strategies. As long as everyone uses monotone strategies, it is a weakly undominated strategy to follow his/her own optimal stopping rule sincerely.